Scattering of non-relativistic finite-size particles and puffy dark matter direct detection

Wu-Long Xu

Chengdu Technological University

Collaborator: Jin min Yang (Institute of Theoretical Physics,
Chinese Academy of Sciences;
Henan Normal University)
Jun Zhao (Henan University)

2026.7.22@紫金山暗物质研讨会 2026

CONTENT

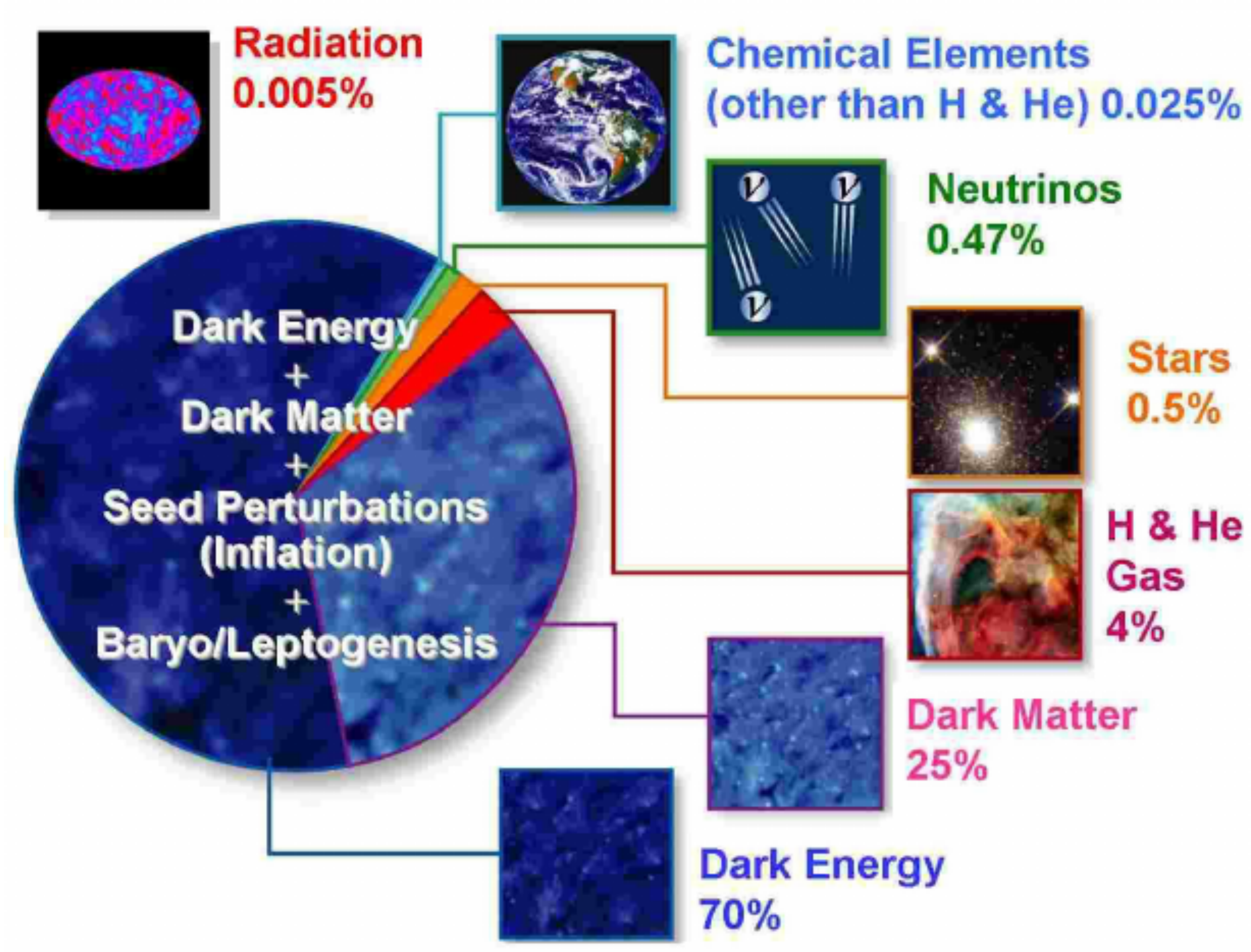
- Introduction to puffy SIDM
- **Scattering of non-relativistic finite-size particles**

  1. Validity of Born Approximation

  2. The regime of scattering cross section
- **Size effect in Puffy dark matter detection**
- Conclusion

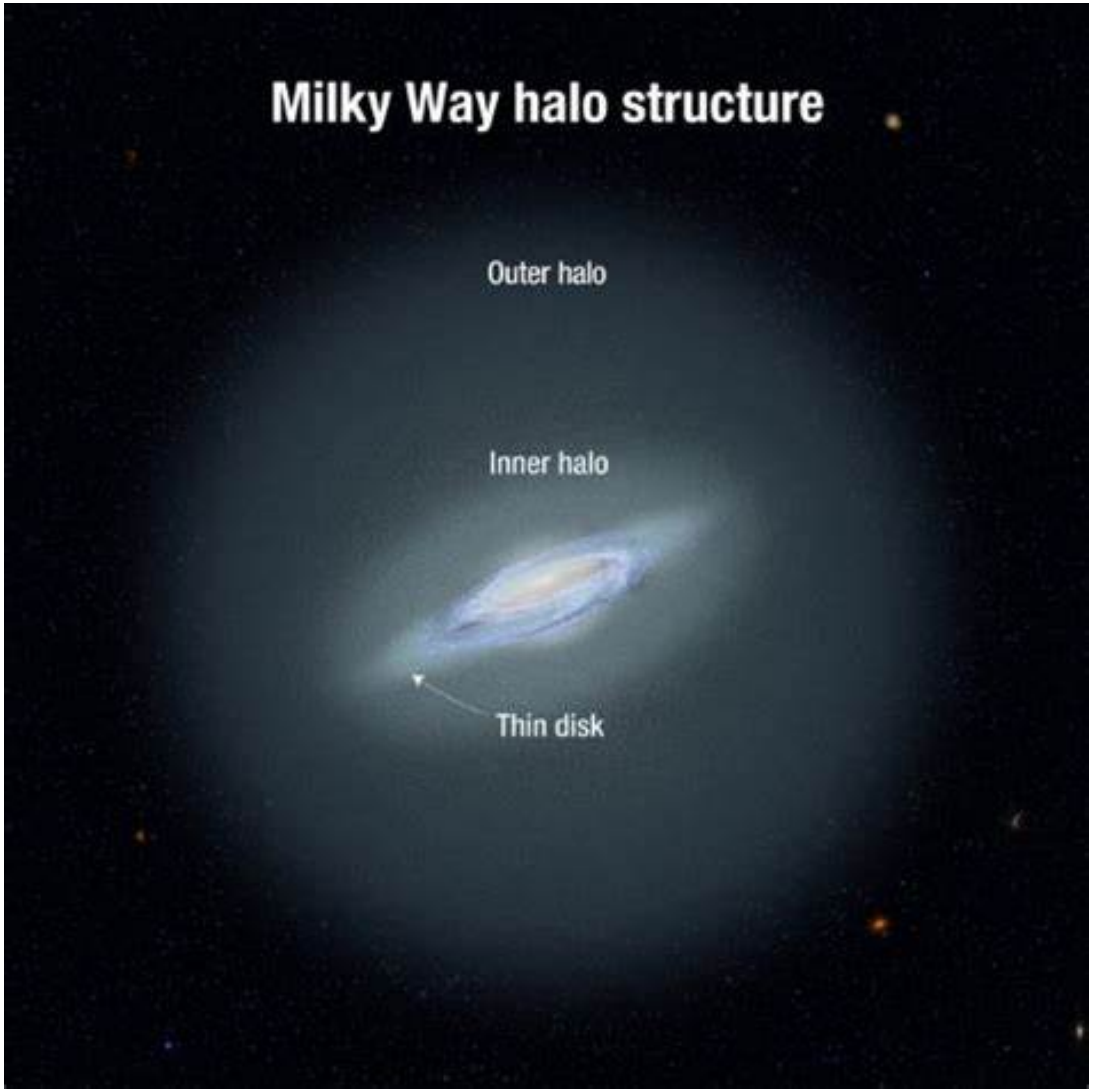
Based on
Wu-Long Xu, Jin Min Yang, Jun Zhao, JHEP04（2026）006

ΛCDM model



Cosmological data has converged upon the ΛCDM paradigm as the standard model of cosmology Of the total mass–energy content of the Universe, approximately 26% is cold dark matter (CDM) and 5% is baryonic matter (while the remainder is consistent with a cosmological constant Λ), with a nearly scale-invariant spectrum of primordial fluctuations. In addition, the ΛCDM model also explains many important aspects of galaxy formation. Figure is taken from Arxiv: 0709.3102.

Problems of CDM in small cosmological structure



1. Cusp vs. Core
2. Too big to fail
3. Missing satellite
4. Diversity problem

ΛCDM

mass deficit

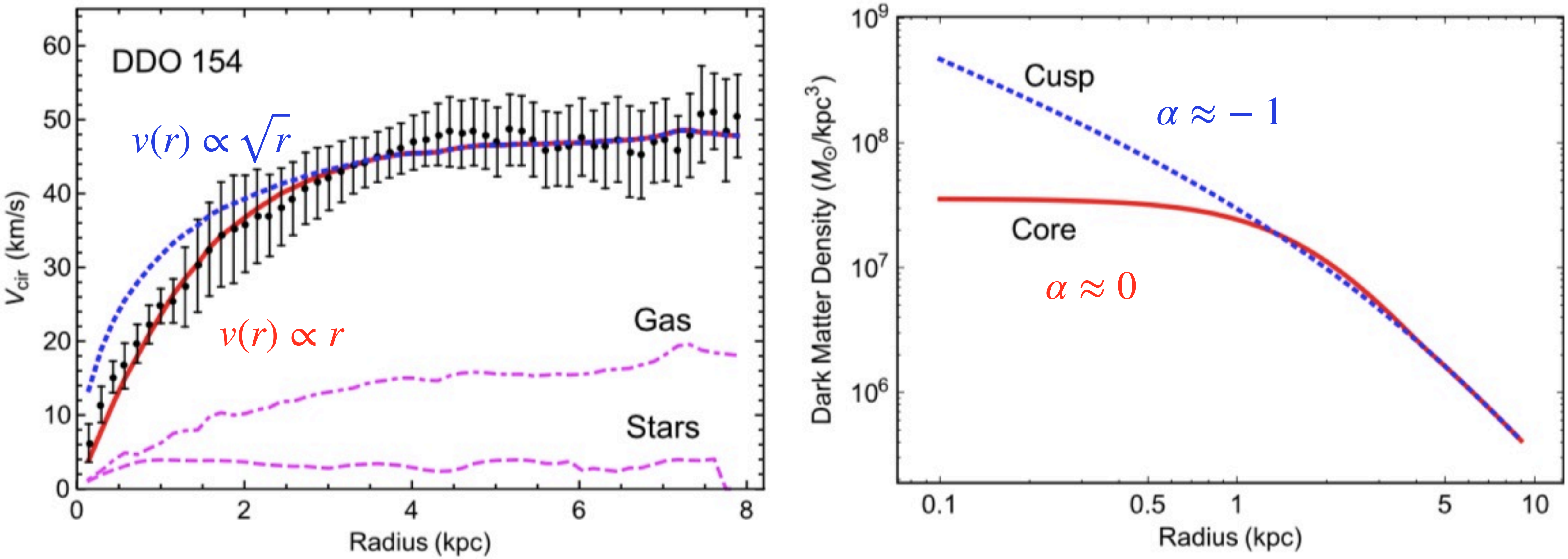
SIDM

smaller central densities

Cusp-Core problem

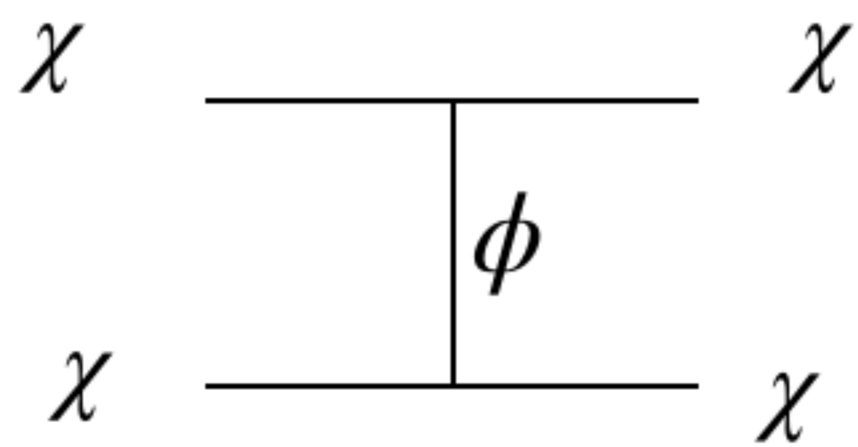
Phys.Rept. 730 (2018) 1-57

$\alpha = d \log \rho_{\mathrm{DM}} / d \log r$



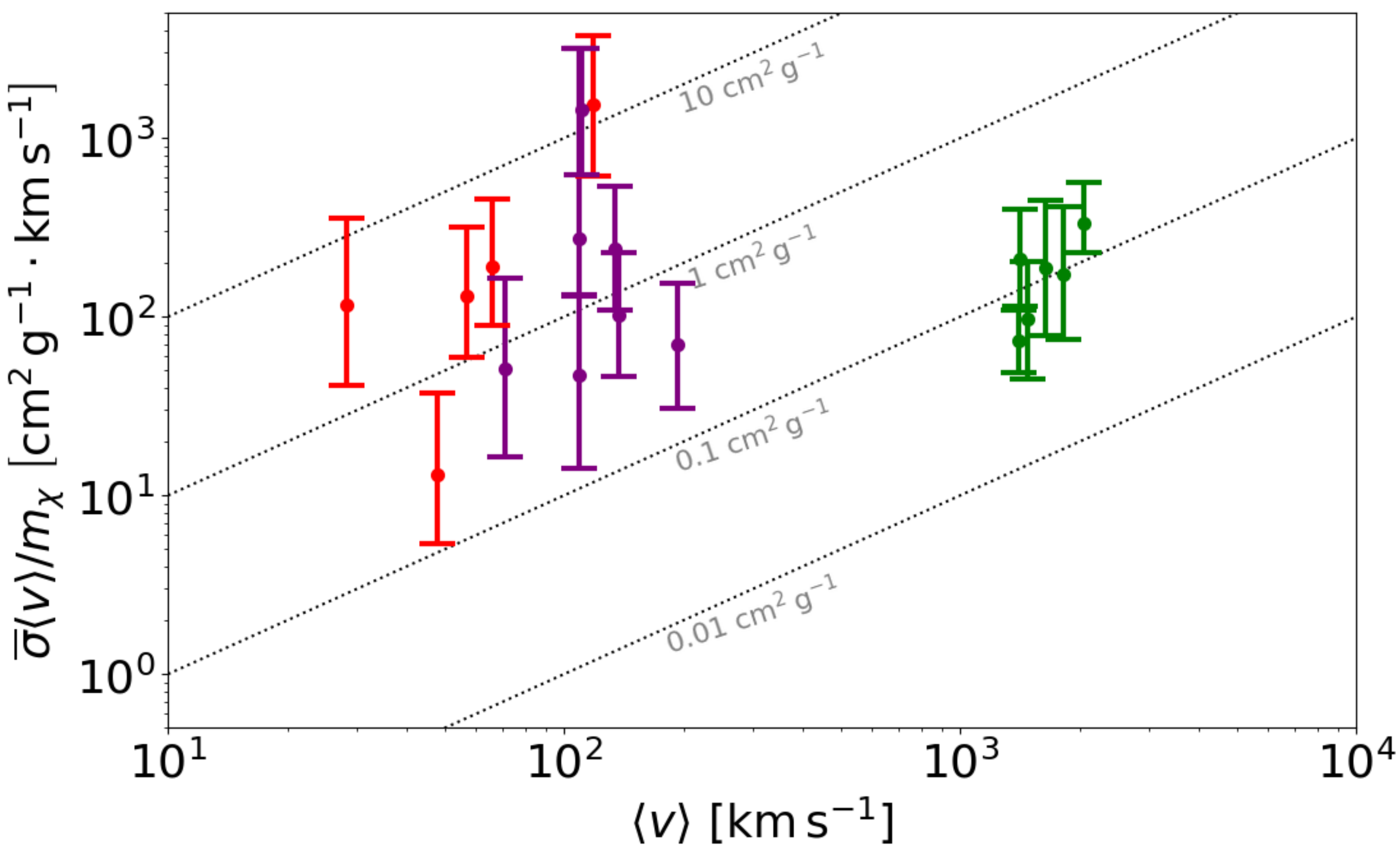
Left: Observed rotation curve of dwarf galaxy DDO 154 (black data points) compared to models with an NFW profile (dotted blue) and cored profile (solid red). Stellar (gas) contributions indicated by pink (dot-)dashed lines. Right: Corresponding DM density profiles adopted in the fits. NFW halo parameters are $r_s \approx 3.4$ kpc and $\rho_s \approx 1.5 \times 10^7\ M_\odot/\mathrm{kpc}^3$, while the cored density profile is generated using an analytical SIDM halo model developed in arXiv:1508.03339,1611.02716.

From astrophysics to particle theory



σ /m depends on the underlying DM particle physics model.

Self-interaction DM



Six clusters form(green)， seven low-surface-brightness spiral galaxies(purple) and five dwarf galaxies of the HI Nearby Galaxy Survey sample(red)

Self-interacting DM candidates

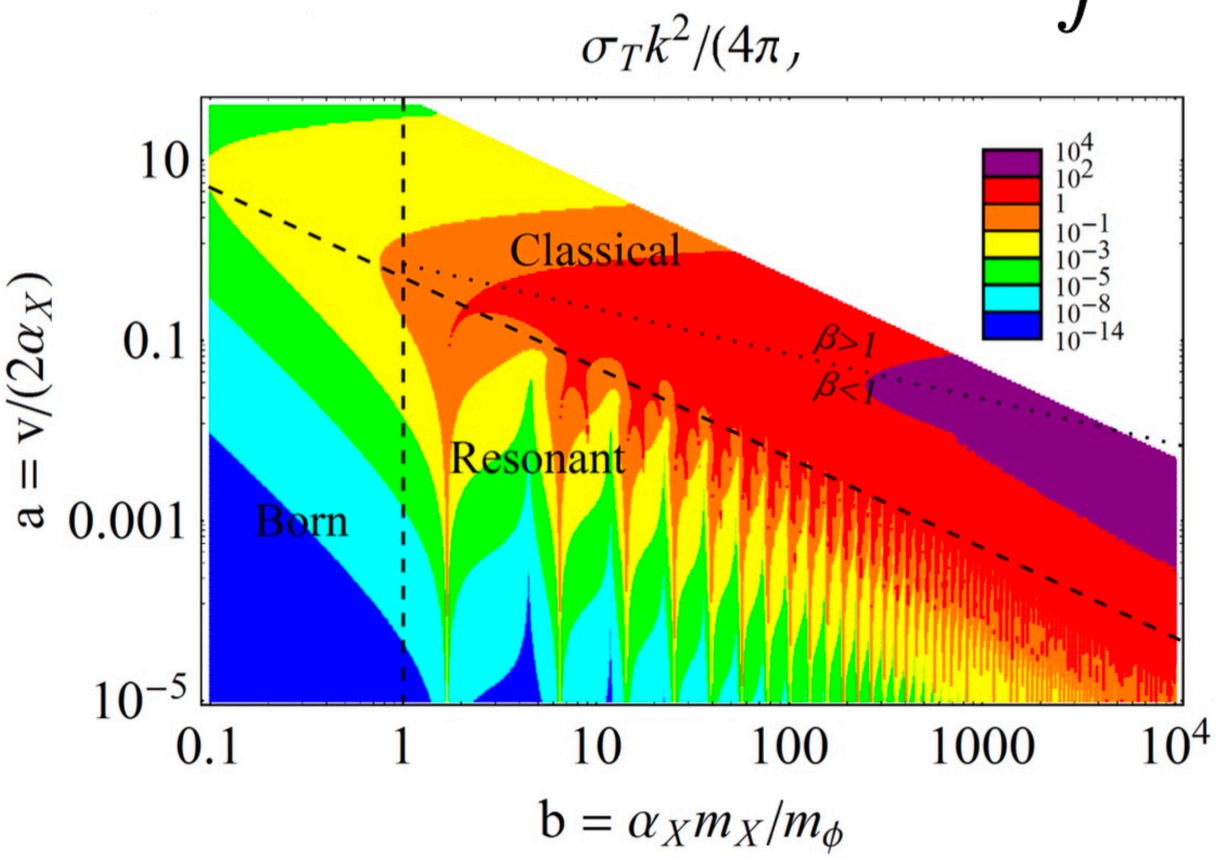
- **Elastic SIDM model**
  - **Light mediator (partial wave analysis)** → **non-relativistic effect**
  - **Composite DM**
    - **Atom DM**
    - **meson, nucleus, Bound state, puffy DM etc.**
      - **puffy DM** → **Size effect**
  - **Resonant SIDM**
    - **Point particle DM**
    - **Dark QCD DM**
- **Inelastic SIDM model**
  - **Two mass splitting dark states**
  - **Excited state**

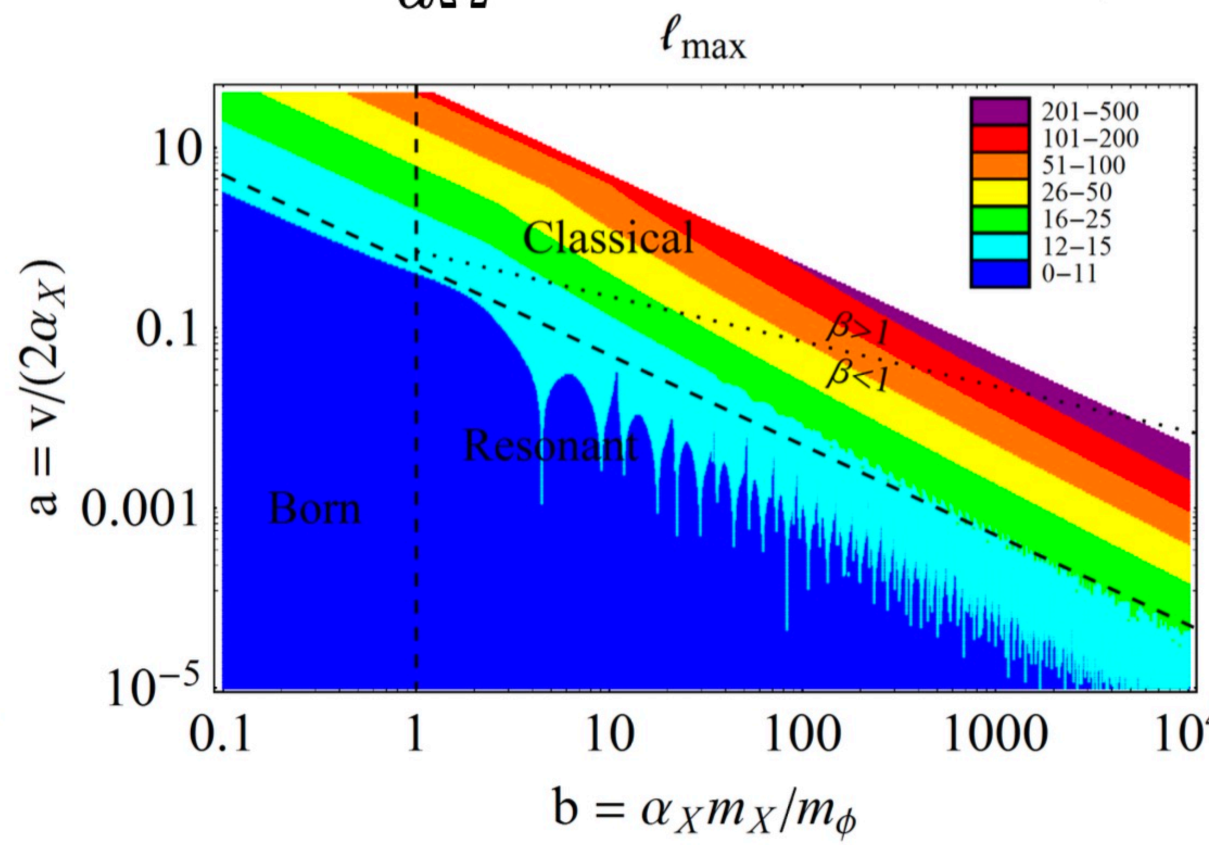
The impact of non-relativistic effects in DM self-scattering

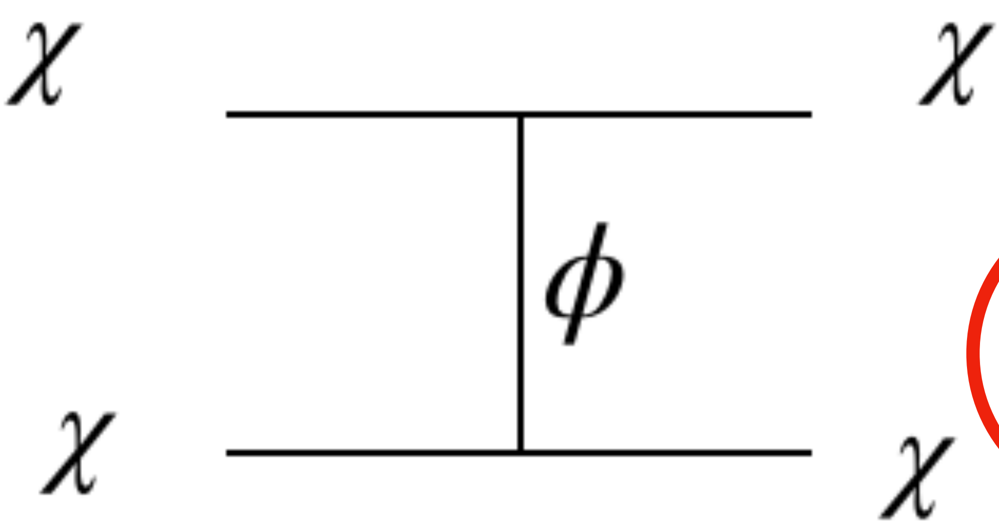
Case of two point dark matter

$$\sigma_T = \int d\Omega\,(1 - \cos\theta)\,\frac{d\sigma}{d\Omega}$$

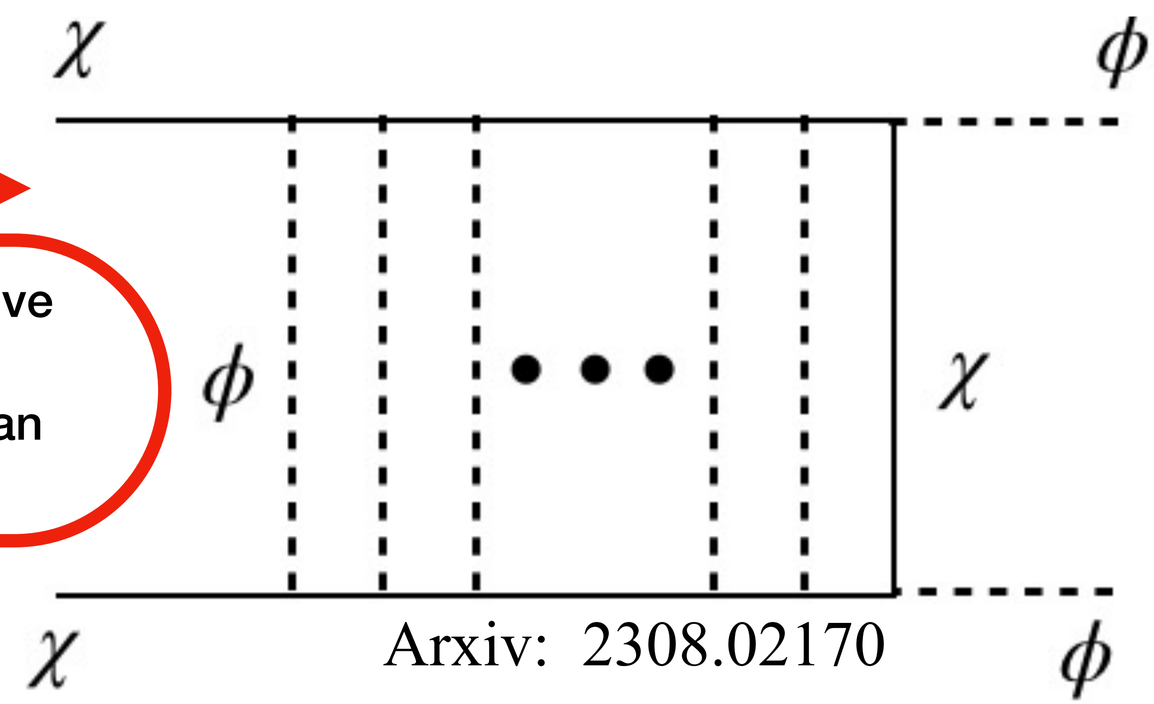
Arxiv: 1302.3898







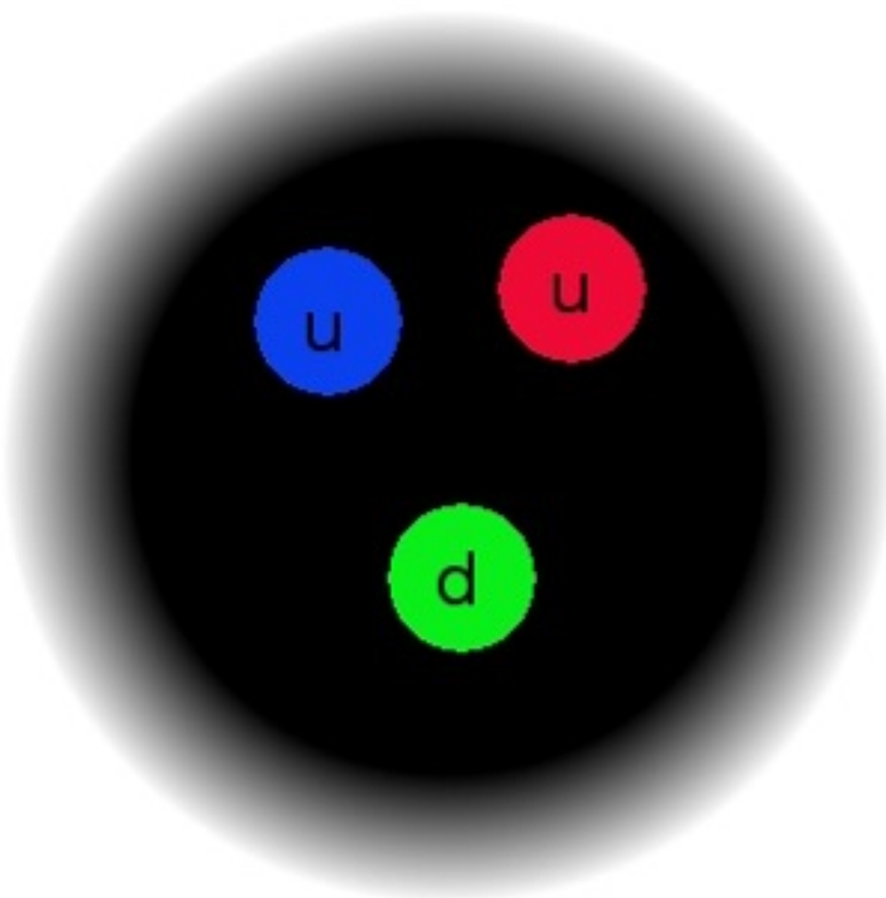
Non-perturbative effect
ladder Feynman diagrams



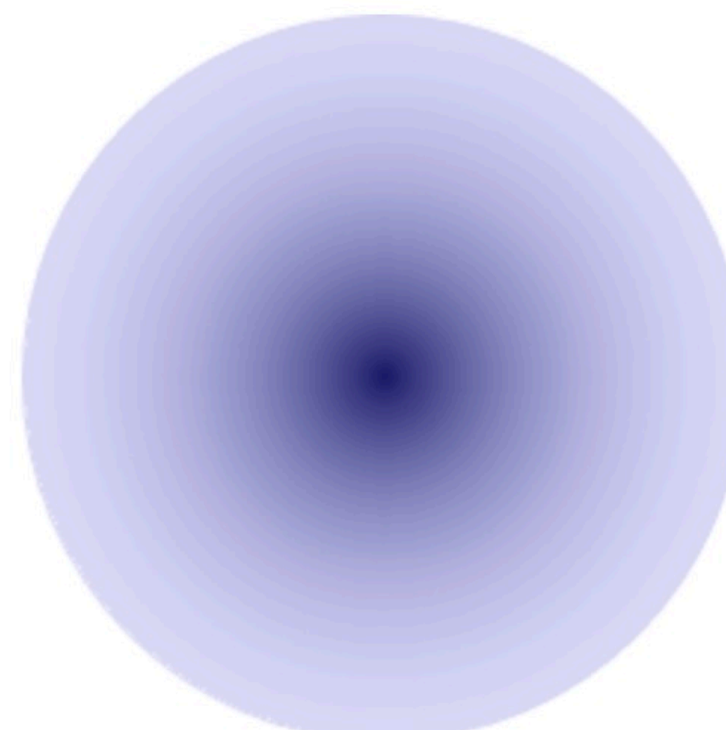
Arxiv: 2308.02170

The impact of non-relativistic finite-size in DM self-scattering

Dark matter has a finite size



Proton



Puffy Dark matter

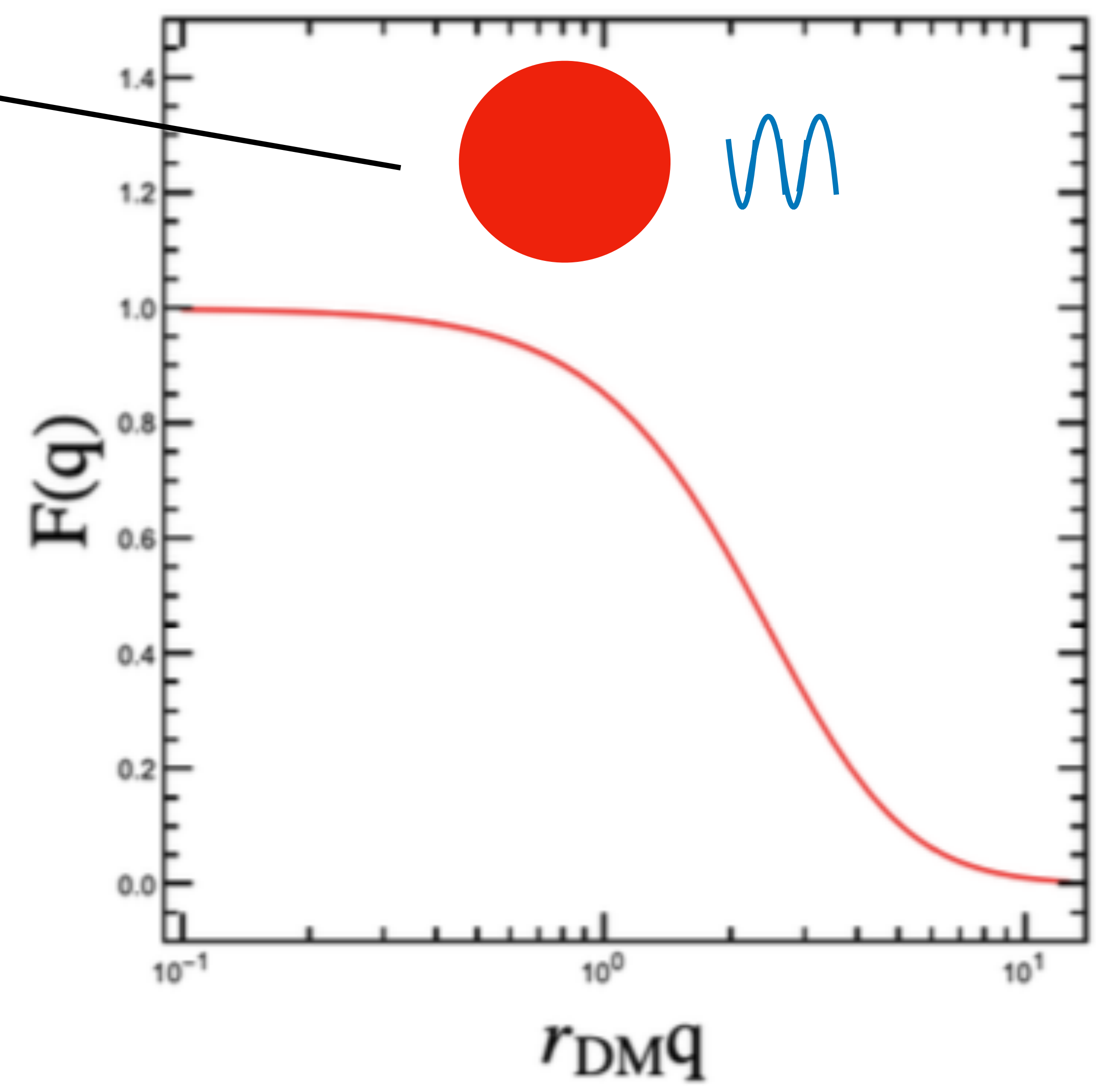
Puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity. The radius effect of DM from the form factor is another source of the velocity of dependence for cross section.

Dipole

$$F(\mathbf{q}) = \int \mathbf{d r} e^{\mathbf{i q r}} \rho(\mathbf{r})$$

$$F(q) = \frac{1}{(1 + 1/24 r_{DM}^2 q^2)^2}$$



- X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

In case of Born approximation

$$V(r) = \frac{Qq}{4\pi\Delta r} = \int \frac{Q\rho(r)}{4\pi|\mathbf{r}-\mathbf{r}'|} d^3r$$

$$M_{fi} = <\psi_f||\psi_i> = \int e^{-ip_3r} V(r) e^{ip_1r} d^3r \quad = \int\int e^{i(p_1-p_3)(r-r')} \frac{Q\rho(r')}{4\pi|\mathbf{r}-\mathbf{r}'|} e^{iqr'} d^3r' d^3r$$

$$= \int e^{iqR} \frac{Q}{4\pi|\mathbf{r}-\mathbf{r}'|} d^3R \int \rho(r') e^{iqr'} d^3r' \quad = (M_{fi})_{point} F(q^2)$$

$$V(r)' = \int \frac{\alpha}{r} e^{\frac{-r}{\lambda}} \rho_1(r_1) dr_1^3 \rho_2(r_2) dr_2^3 \qquad M_{fi} = \frac{4\pi\alpha}{\vec{q}^{\,2} + \lambda^{-2}} F_1(q) F_2(-q)$$

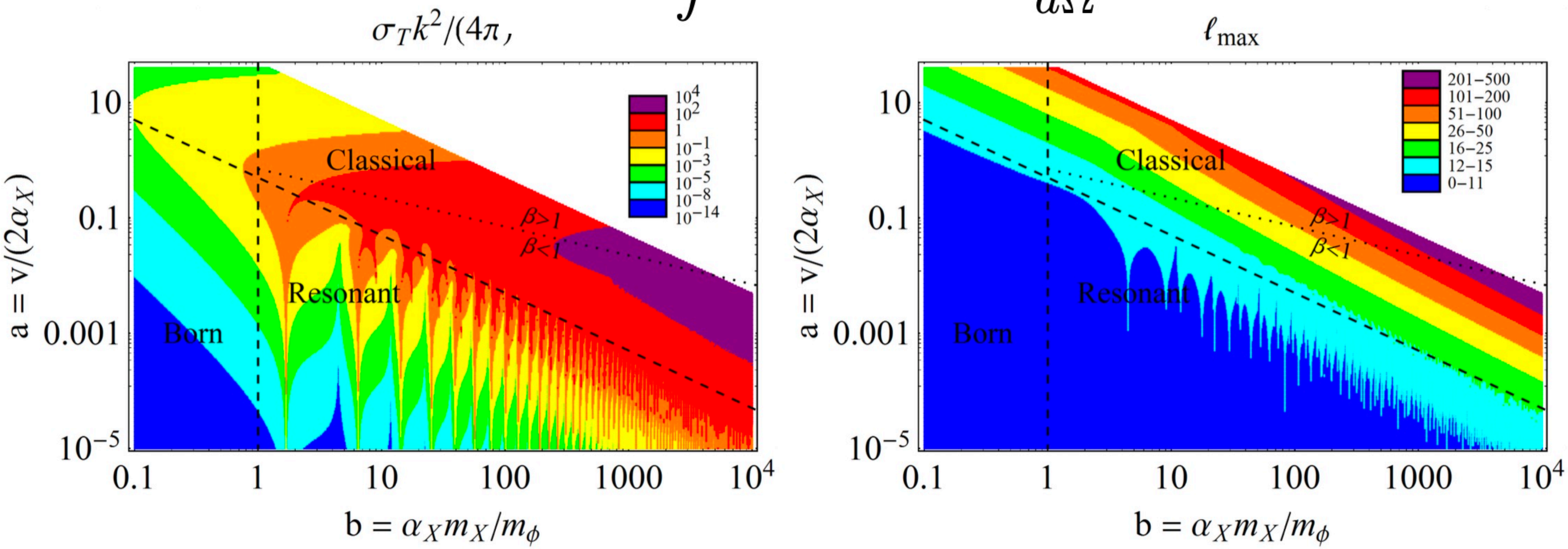
$$r_{DM}^2 = \int d\mathbf{r} \rho(r) r^2 = -6 \frac{d^2F(q)}{dq^2}\Big|_{q=0}$$

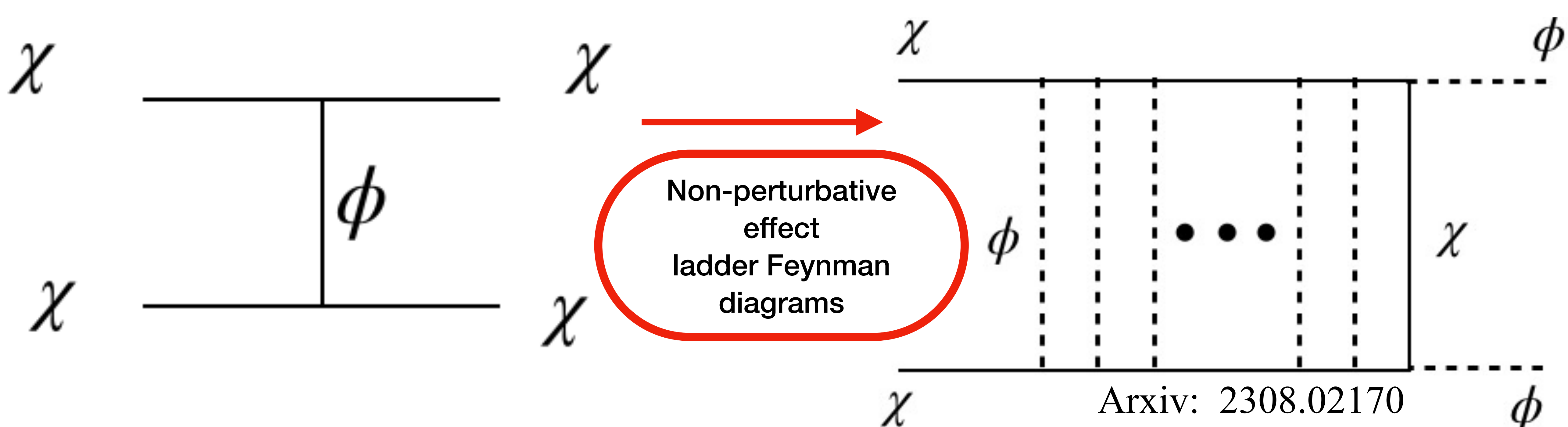
$$F(q^2) = \int \rho(r') e^{iqr'} d^3r' = \frac{4\pi}{q} \int_0^\infty r \sin(qr) \rho(r) dr = Q - \frac{1}{3!} < R^2 > q^2 + \cdots$$

Case of two point dark matter

$$\sigma_T = \int d\Omega\,(1-\cos\theta)\,\frac{d\sigma}{d\Omega}$$

Arxiv: 1302.3898





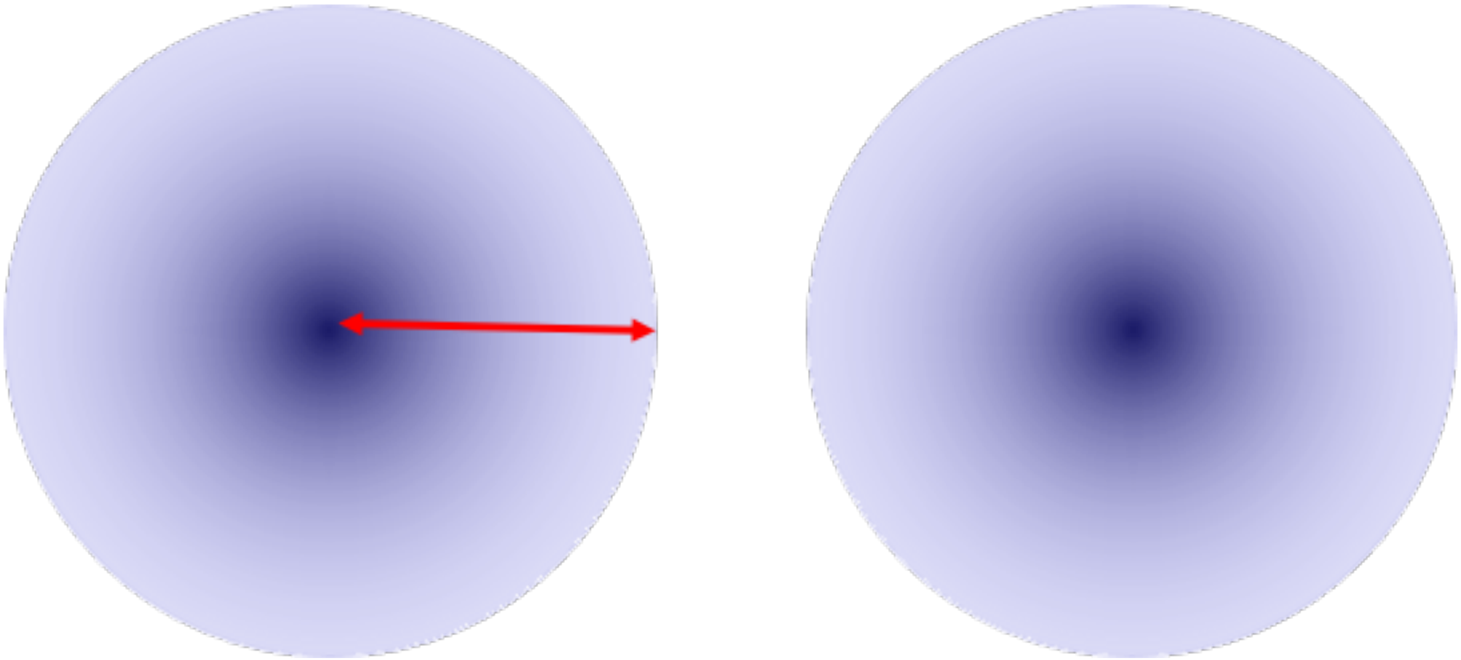
Arxiv: 2308.02170

Two points need to be considered on

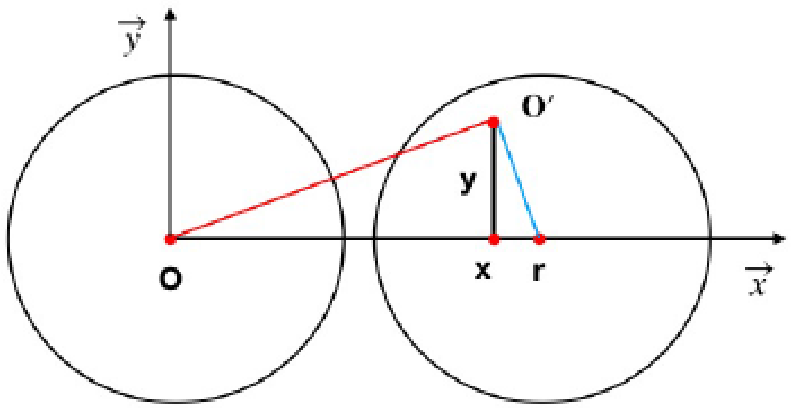
- **The validity of Born approximation**
- **The regime of scattering cross section**

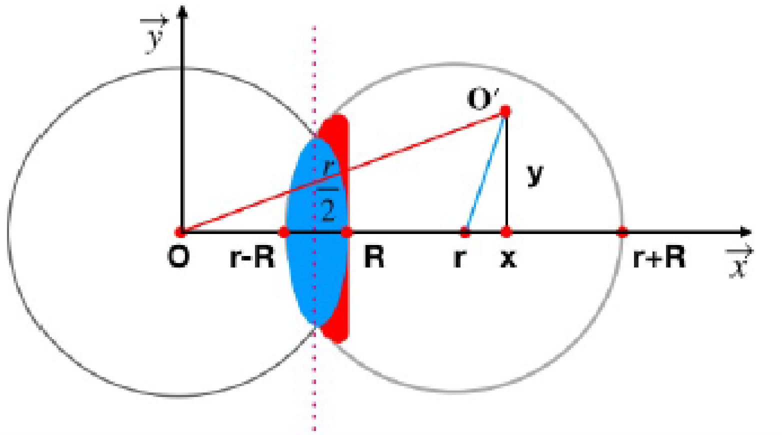
The validity of Born approximation

Case of finite-size particles



$$\rho(r) = \frac{3Q}{4\pi R_\chi^3}\theta(R_\chi - r),$$





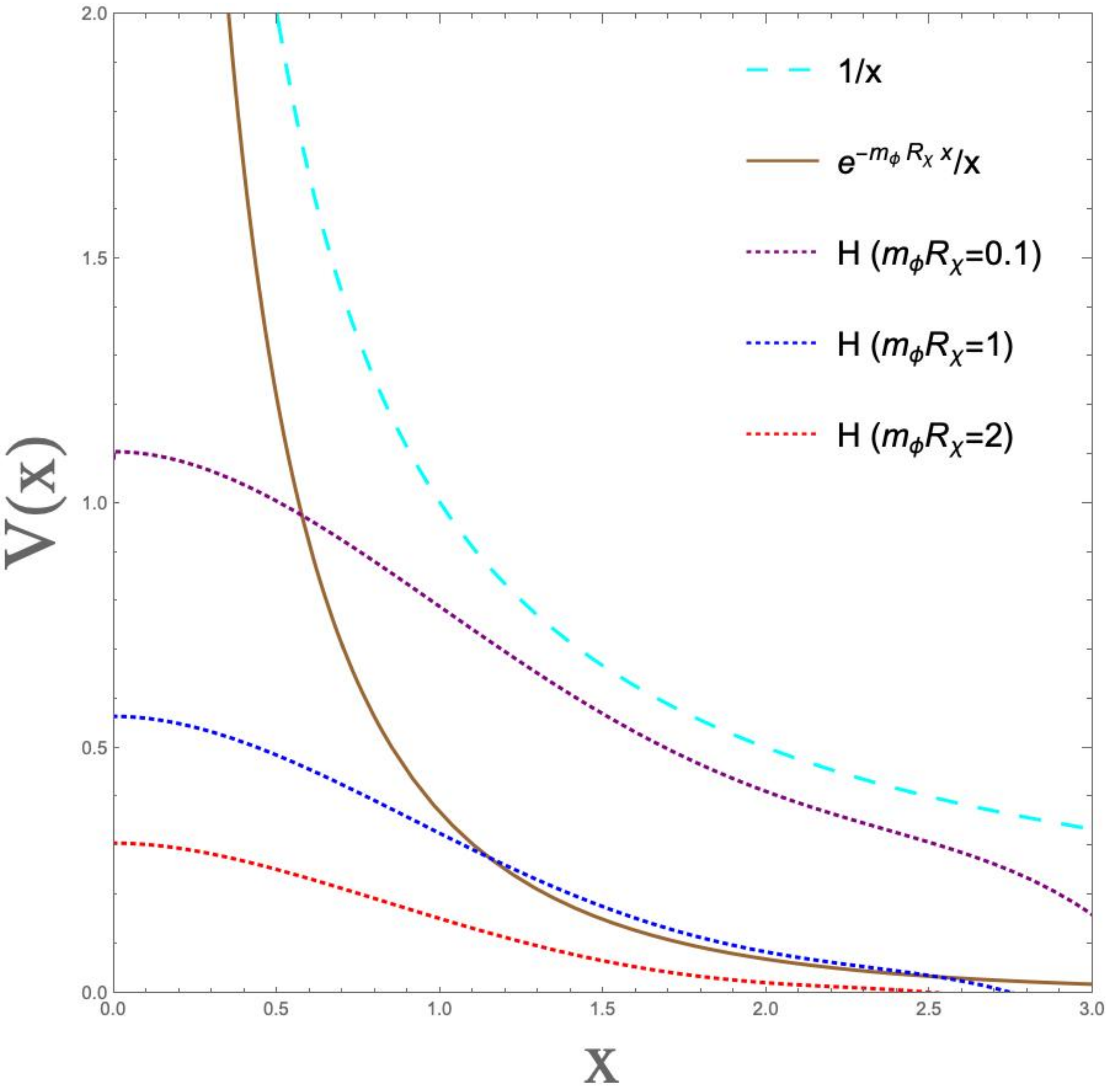
$$V_{\chi-\mathrm{N}}(r) = \begin{cases} g(r, R_\chi, R_N) & \text{for } r < 2R_\chi \\ \alpha \dfrac{e^{-m_\phi r}}{r} h\left(R_\chi, R_N\right) & \text{for } r > 2R_\chi \end{cases}$$

$$\begin{aligned} g(r, R_\chi, R_N) = \frac{3\alpha}{16\pi m_\phi^6 R_\chi^3 R_N^3} \Big\{ & m_\phi^4 (r - 2R_\chi)^2 (r + 4R_\chi) + \frac{6}{r} \Big[-e^{m - \phi(r + R_\chi + R_N)} \\ & \times (-2 - 2m_\phi R_\chi + e^{m_\phi R_\chi}(2 + m_\phi R_\chi(-2 + m_\phi(r - 2R_\chi)) + 2m_\phi R_\chi)) \\ & \times (1 + m_\phi R_N + e^{2m_\phi R_N}(-1 + m_\phi R_N)) \\ & + e^{-m_\phi R_N}(1 + m_\phi R_N)(2(2 + m_\phi^2 r(r - 2R_\chi)) \cosh(m_\phi R_\chi) \\ & - 4(\cosh[m_\phi(r - R_\chi)] + m_\phi R_\chi \sinh[m_\phi(r - R_\chi)]) \\ & + 4m_\phi(r - R_\chi) \sinh(m_\phi R_\chi))\Big]\Big\} \end{aligned}$$

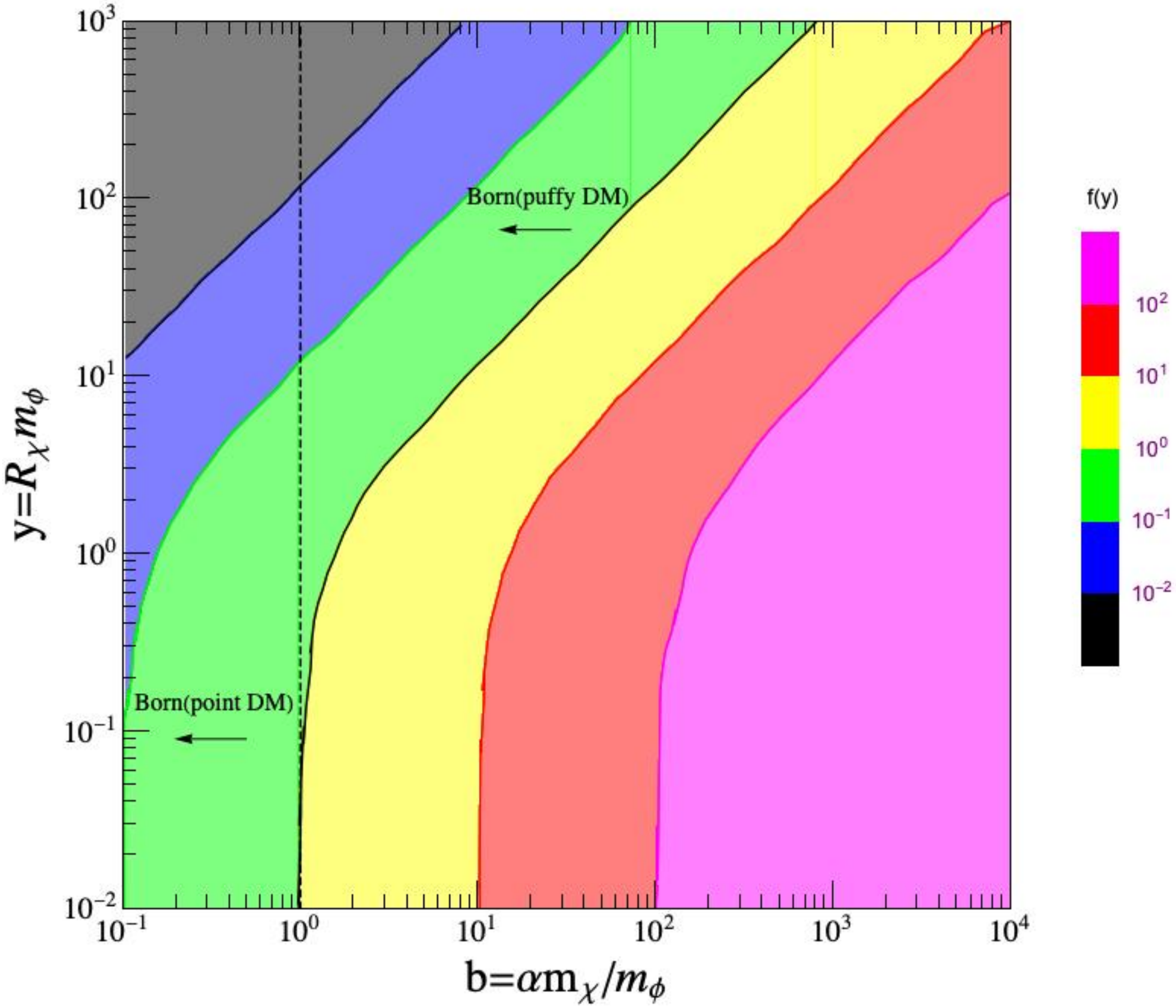
$$\begin{aligned} h\left(R_\chi, R_N\right) = 4\pi \left(\frac{3}{4\pi R_\chi^3}\right) \left(\frac{3}{4\pi R_N^3}\right) \frac{\pi}{m_\phi^2} & \left(\frac{e^{-m_\phi R_\chi}}{m_\phi^2} - \frac{e^{m_\phi R_\chi}}{m_\phi^2} + \frac{R_\chi e^{-m_\phi R_\chi}}{m_\phi} + \frac{R_\chi e^{m_\phi R_\chi}}{m_\phi}\right) \\ \times & \left(\frac{e^{-m_\phi R_N}}{m_\phi^2} - \frac{e^{m_\phi R_N}}{m_\phi^2} + \frac{R_N e^{-m_\phi R_N}}{m_\phi} + \frac{R_N e^{m_\phi R_N}}{m_\phi}\right) \end{aligned}$$

The validity of Born approximation

1. The Yukawa potential are approaching the Coulomb potential in force range with a pole
2. The puffy potential removes the pole and the strength decrease along with the increase of the radius.



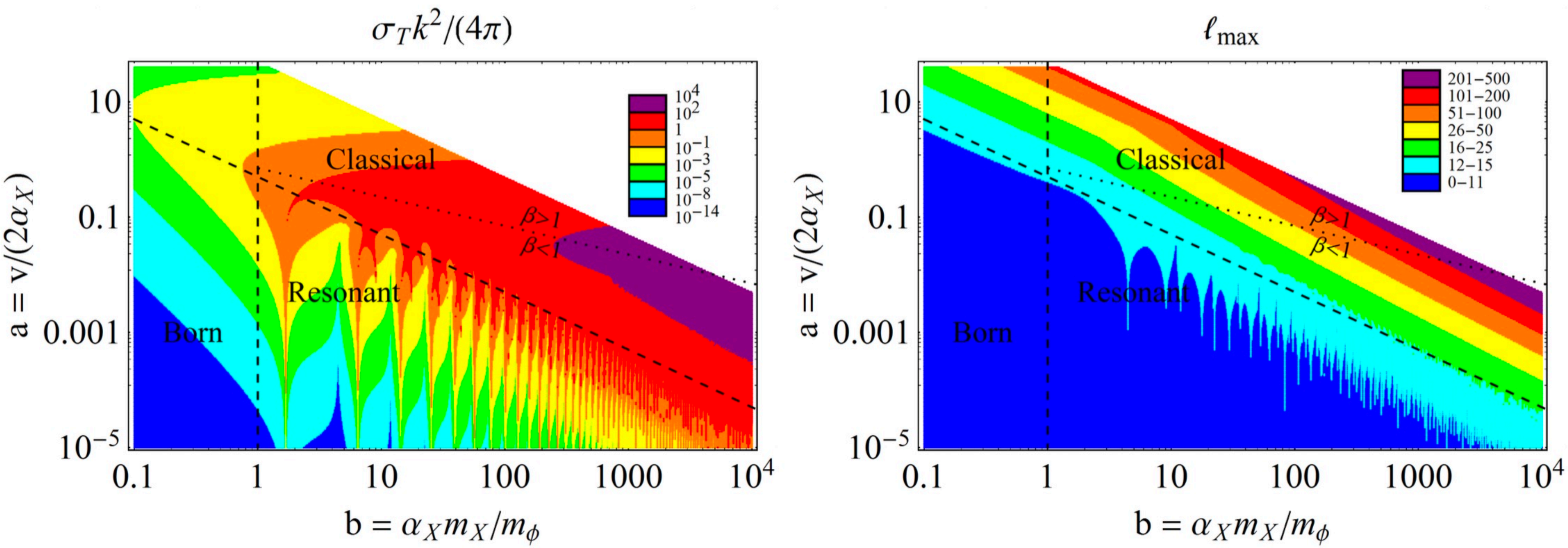
The validity of Born approximation



$$m_\chi \left| \int_0^\infty r V_{\rm puffy}(r) dr \right| = \frac{\alpha m_\chi}{m_\phi} \frac{3(-15 + y^2(15 - 10y + 4y^3) + 15(1+y^2)e^{-2y})}{10y^6}$$
$$= b f(y) \ll 1 ,$$

The regime of scattering cross section

Point-like dark matter self-scattering



Born $\alpha m_\chi/m_\phi < 1$ **nonperturbative regime** $\alpha m_\chi/m_\phi < 1$

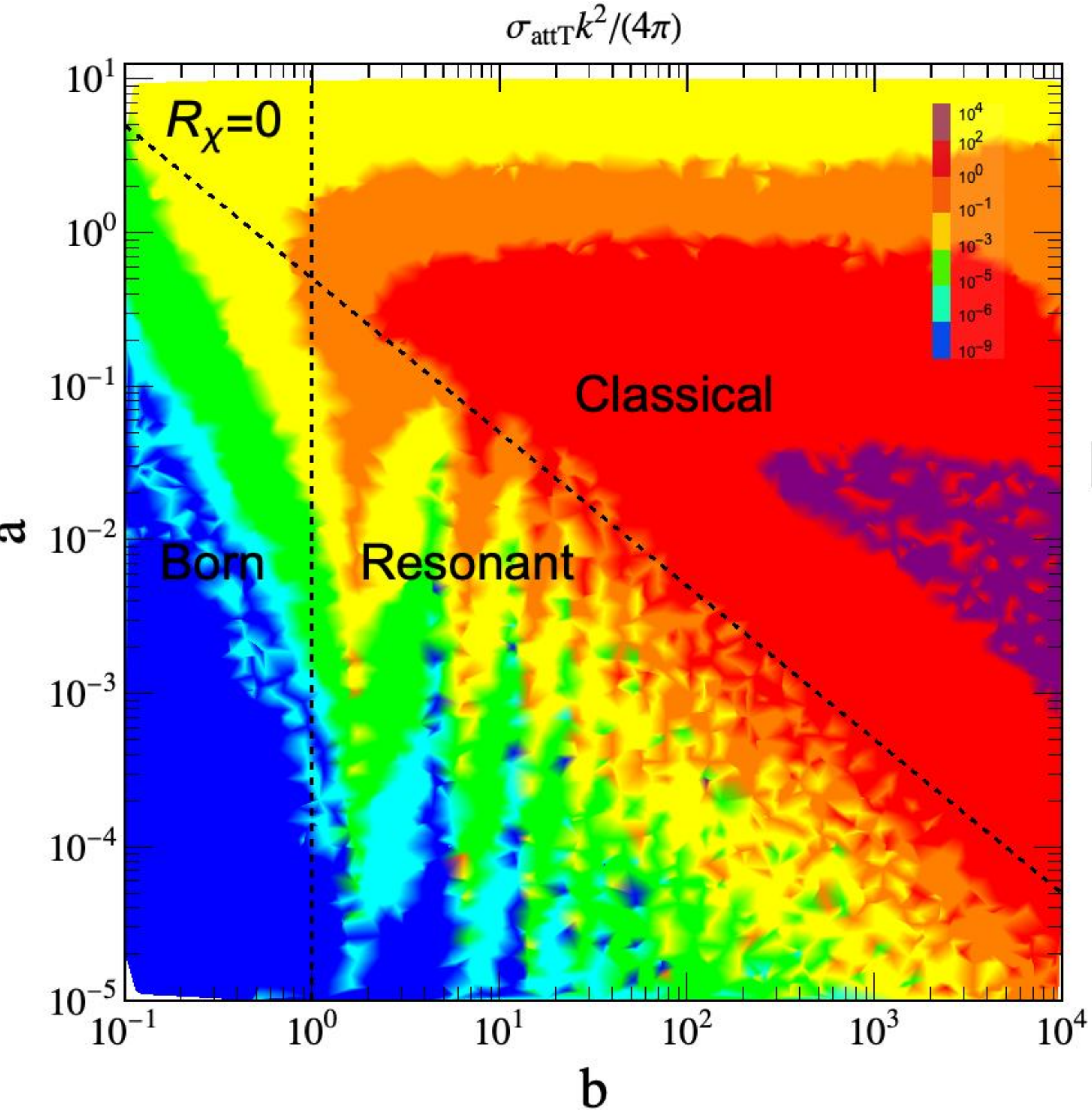
quantum $k/m_\phi \ll 1$ **classical** $k/m_\phi \gg 1$

weakly coupled $2\alpha m_\phi/(m_\chi v^2) \ll 1$ **strongly coupled** $2\alpha m_\phi/(m_\chi v^2) \gg 1$

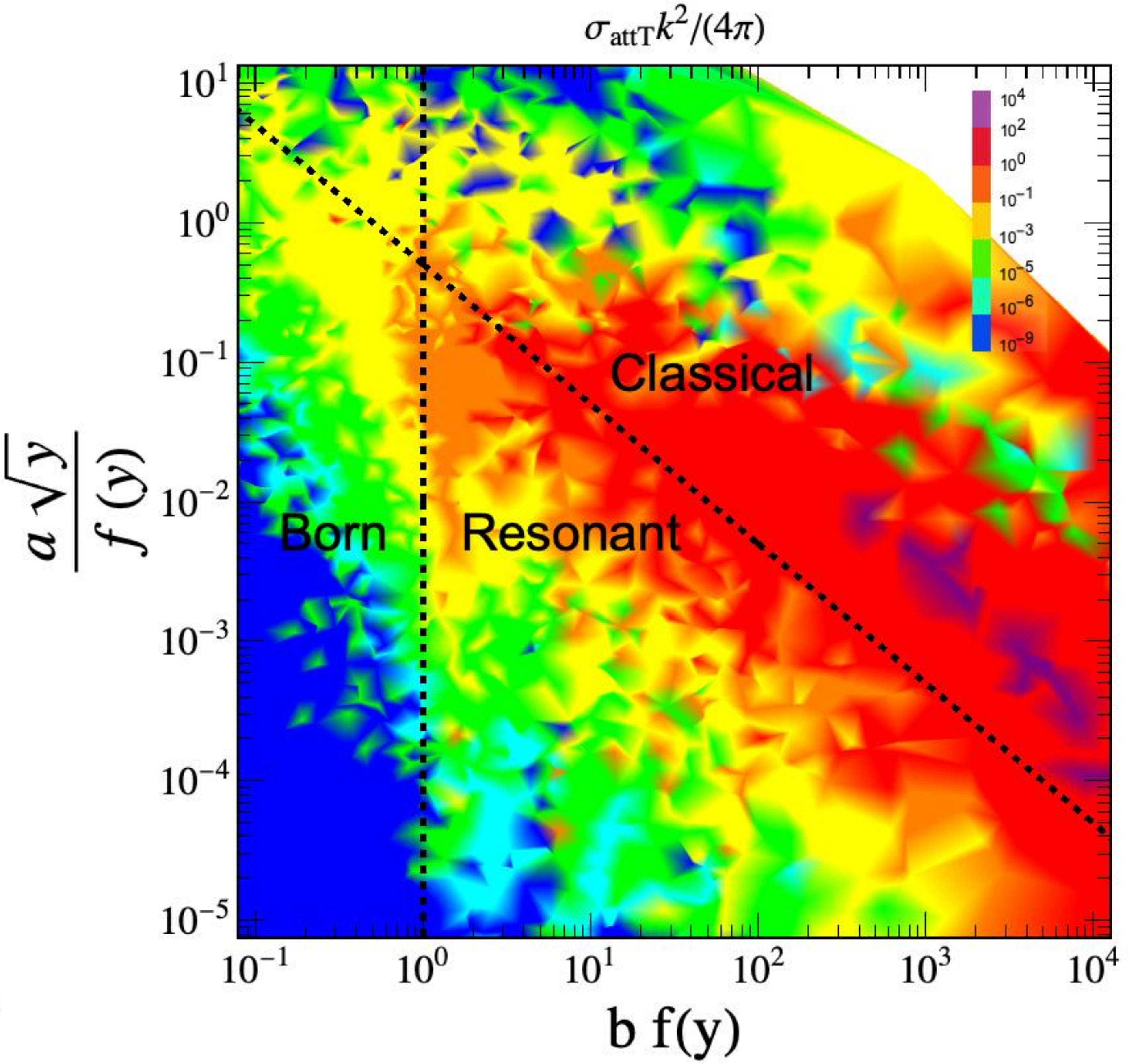
Case of finite-size particles self scattering

	Point	Puffy
Born	$b = \alpha m_\chi / m_\phi < 1$	$bf(y) < 1$
Resonant	$b > 1$ $m_\chi v / m_\phi < 1$	$bf(y) > 1$ $\sqrt{m_\chi v / m_\phi \times m_\chi v R_\chi} < 1$
Classical	$m_\chi v / m_\phi > 1$	$\sqrt{m_\chi v / m_\phi \times m_\chi v R_\chi} > 1$

Case of finite-size particles self scattering



Point-like particles case

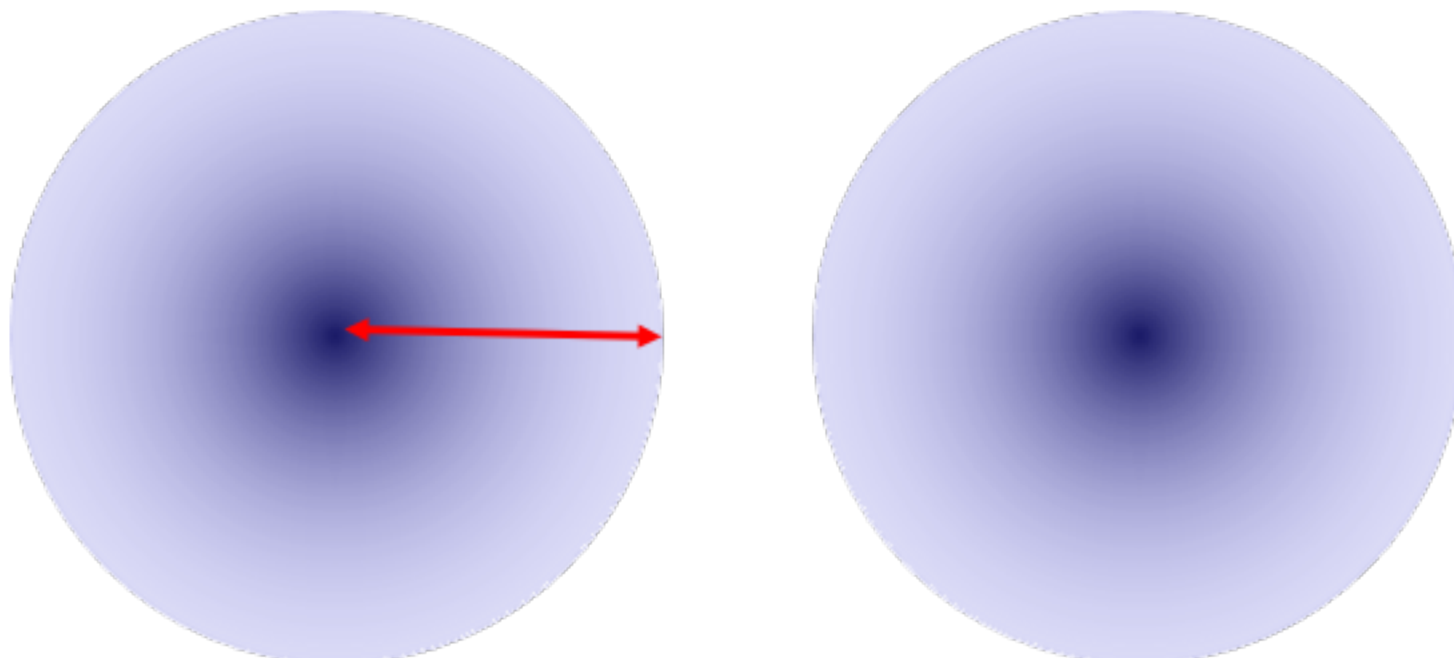


Puffy particles case

What about puffy DM direct detection（the scattering between different finite-size particles ）

The validity of Born approximation

Case of finite-size particles

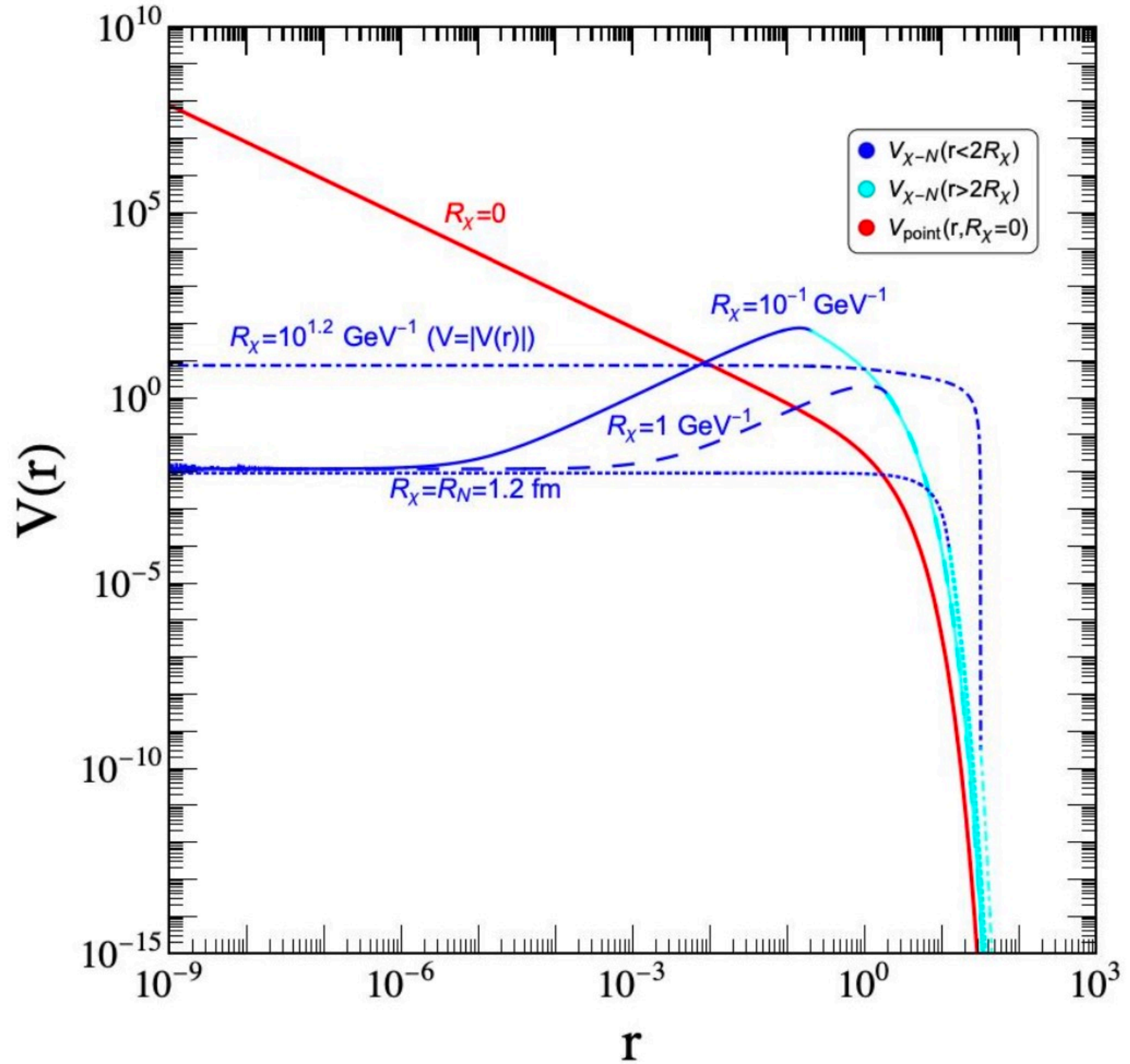


$$\rho(r) = \frac{3Q}{4\pi R_\chi^3}\theta(R_\chi - r),$$

The validity of Born approximation

1.fix the nucleon size a typical value $R_N = 1.2$ fm ~ 6.2 GeV^{-1} and compare several cases with different dark matter radii.

2.For a puffy dark matter particle with a size comparable to or larger than the nucleon size, the potential remains nearly a constant before dropping sharply. While for a dark matter particle size smaller than the nucleon size, the potential first remains flat, then increases slightly and finally drops off as r increases.



The validity of Born approximation

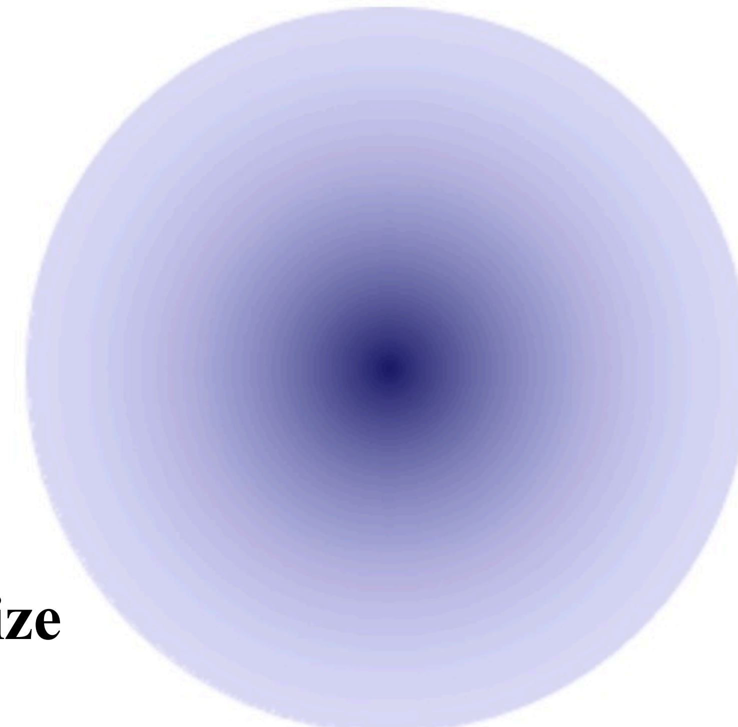
a nucleon with size

$R_N = 1.2$ fm

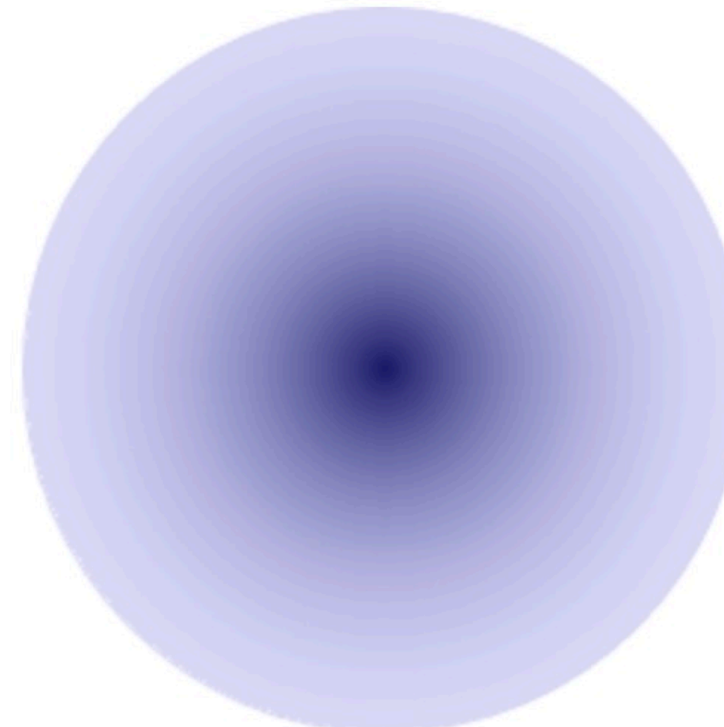
a dark matter particle with size

$R_\chi = 6.2n\ \mathrm{GeV}^{-1}$

$R_\chi / R_N = n$



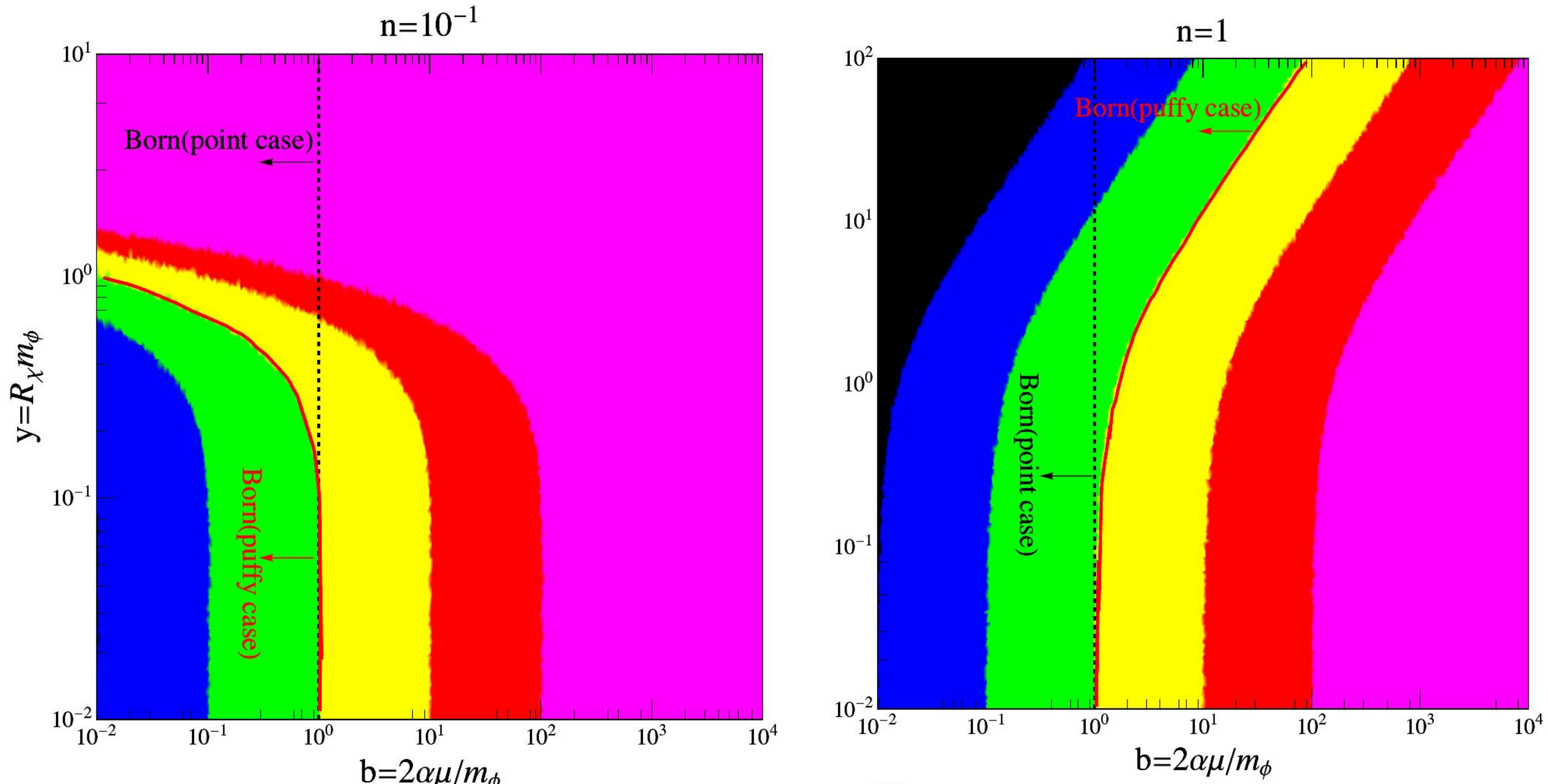
Puffy Dark matter



Nucleon

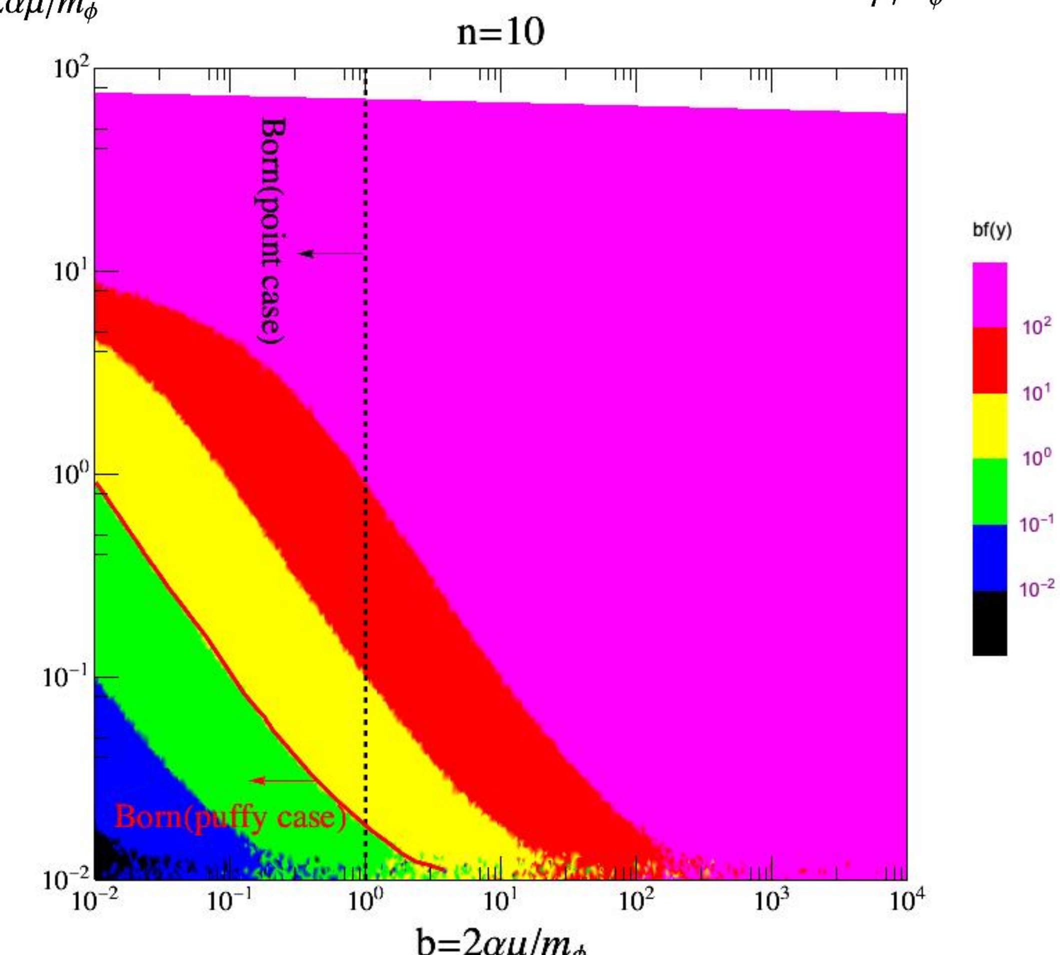
$$V_{\chi-\mathrm{N}}(r) = \begin{cases} g(r, 6.2n\ \mathrm{GeV}^{-1}, 6.2\ \mathrm{GeV}^{-1}) & \text{for } r < 2R_\chi\,, \\ \alpha \frac{e^{-m_\phi r}}{r} \times h(r, 6.2n\ \mathrm{GeV}^{-1}, 6.2\ \mathrm{GeV}^{-1}) & \text{for } r > 2R_\chi\,. \end{cases}$$

The validity of Born approximation



1. parameter space satisfying the Born approximation condition is sensitive to the sizes of the scattering particles.

2. using the tree-level calculation from quantum field theory to obtain scattering cross sections is not accurate in such a case.



Size effect in Puffy dark matter detection

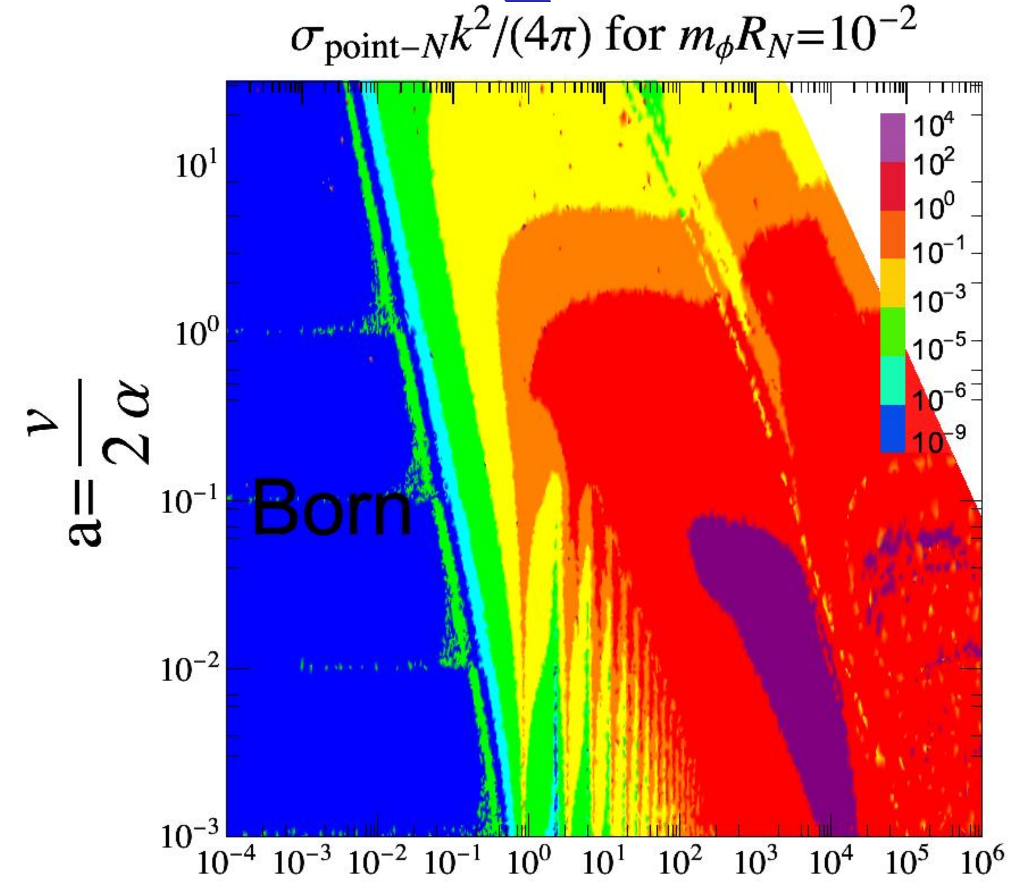
Point-like dark matter-nucleus scattering

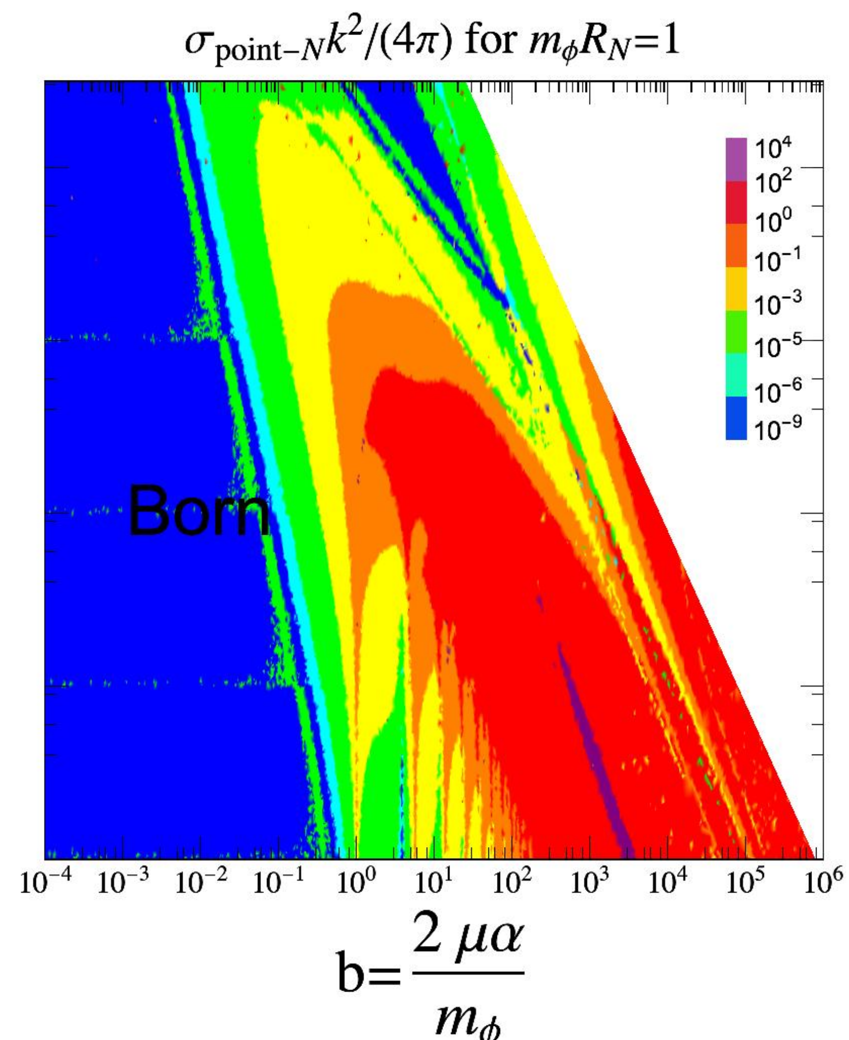
A proton with mass number A = 1

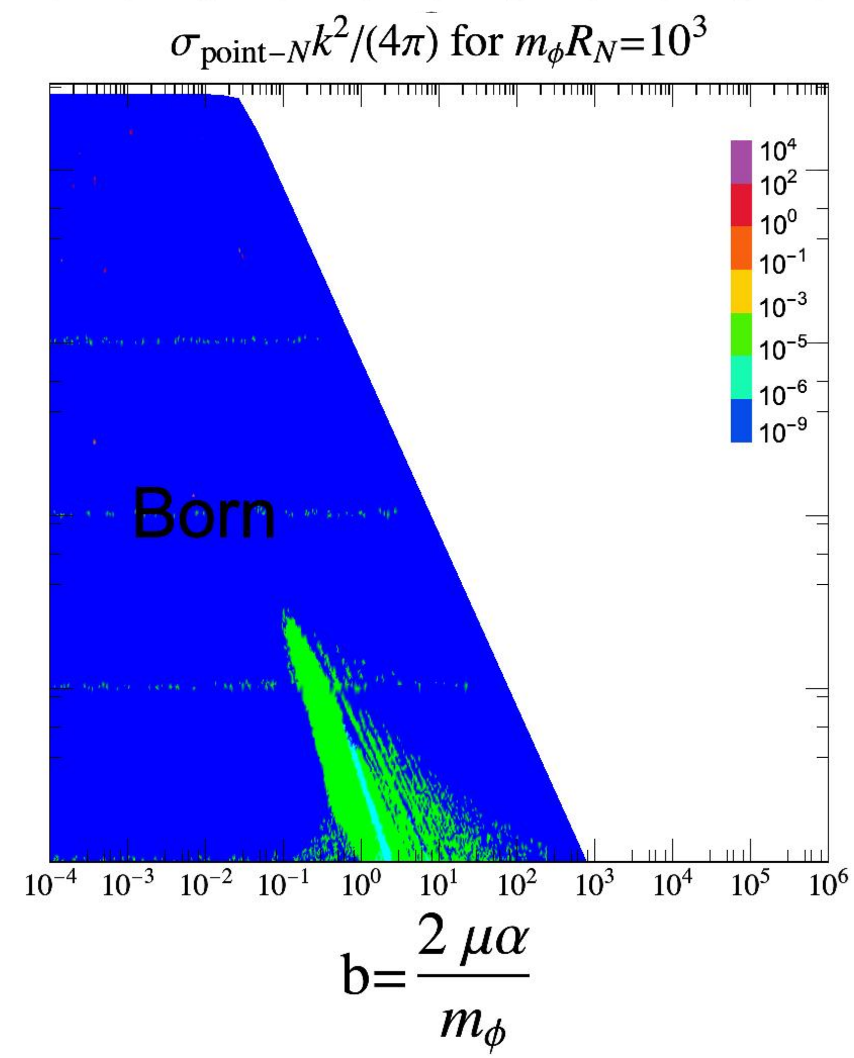
mN = 0.938 GeV

Fix the dark matter velocity as 300 km/s

the finite size of the target nucleus may introduce non-perturbative effects that differ from the scenario of point-like dark matter.







Puffy dark matter-nucleus scattering

Definition $R_\chi/R_N = n$

Proton $R_\chi = 6.2n\text{GeV}^{-1}$

xenon nucleus $R_\chi = 30.67n\text{GeV}^{-1}$

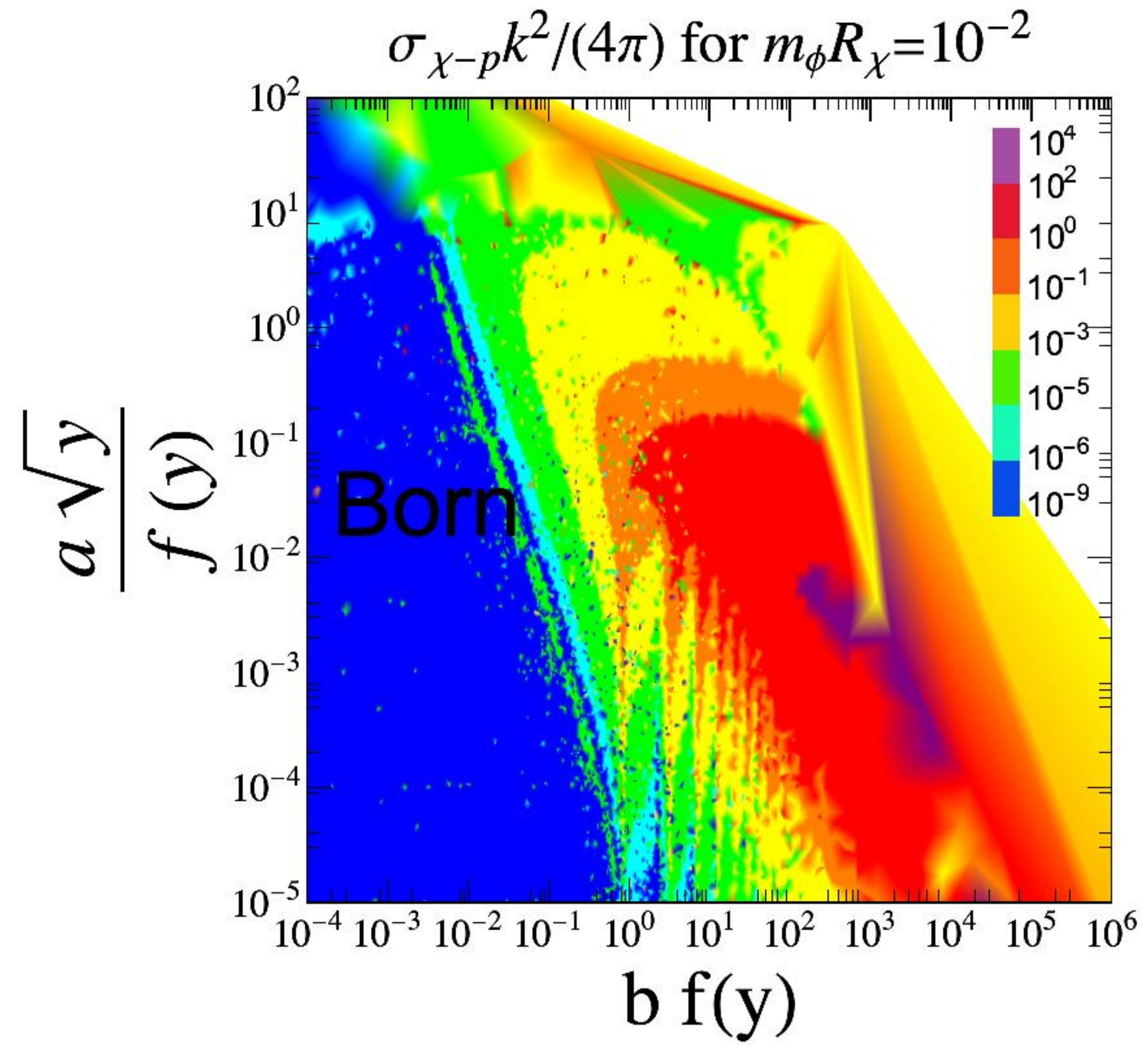
Puffy dark matter-xenon nucleus scattering with nucleon number A = 132

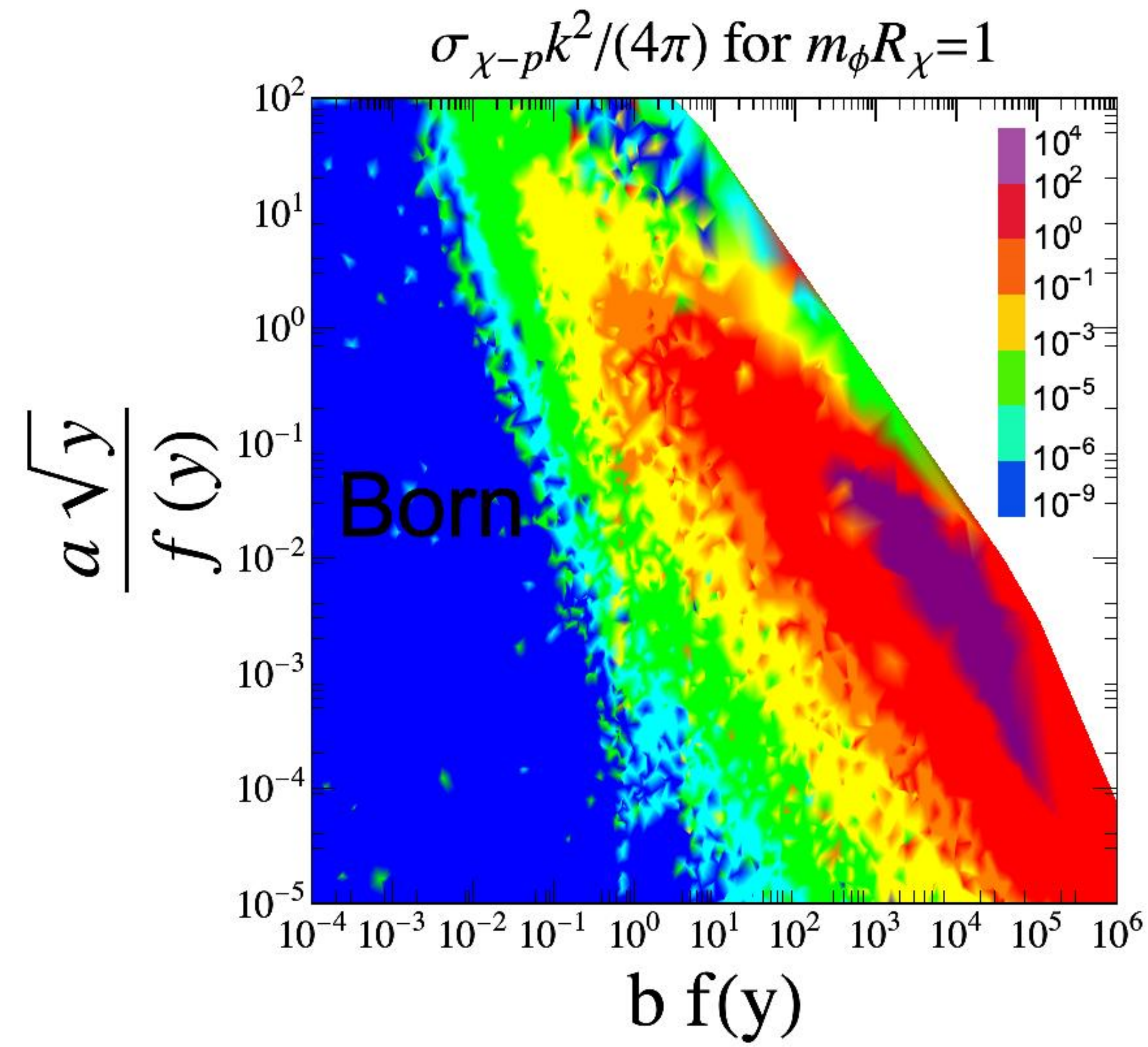
For a given value of y, we have $n = y/(m_\varphi \times 6.2\ \text{GeV}^{-1})$

Parameter space (m_φ, α, M_χ) with M_χ defined by $M_\chi = N m_\chi$

$$V_{\chi-\mathrm{N}}(r) = \begin{cases} g(r, 30.67n\ \mathrm{GeV}^{-1}, 30.67\ \mathrm{GeV}^{-1}) & r < 2R_\chi\,, \\ \alpha\frac{e^{-m_\phi r}}{r} \times h(r, 30.67n\ \mathrm{GeV}^{-1}, 30.67\ \mathrm{GeV}^{-1}) & r > 2R_\chi\,. \end{cases}$$

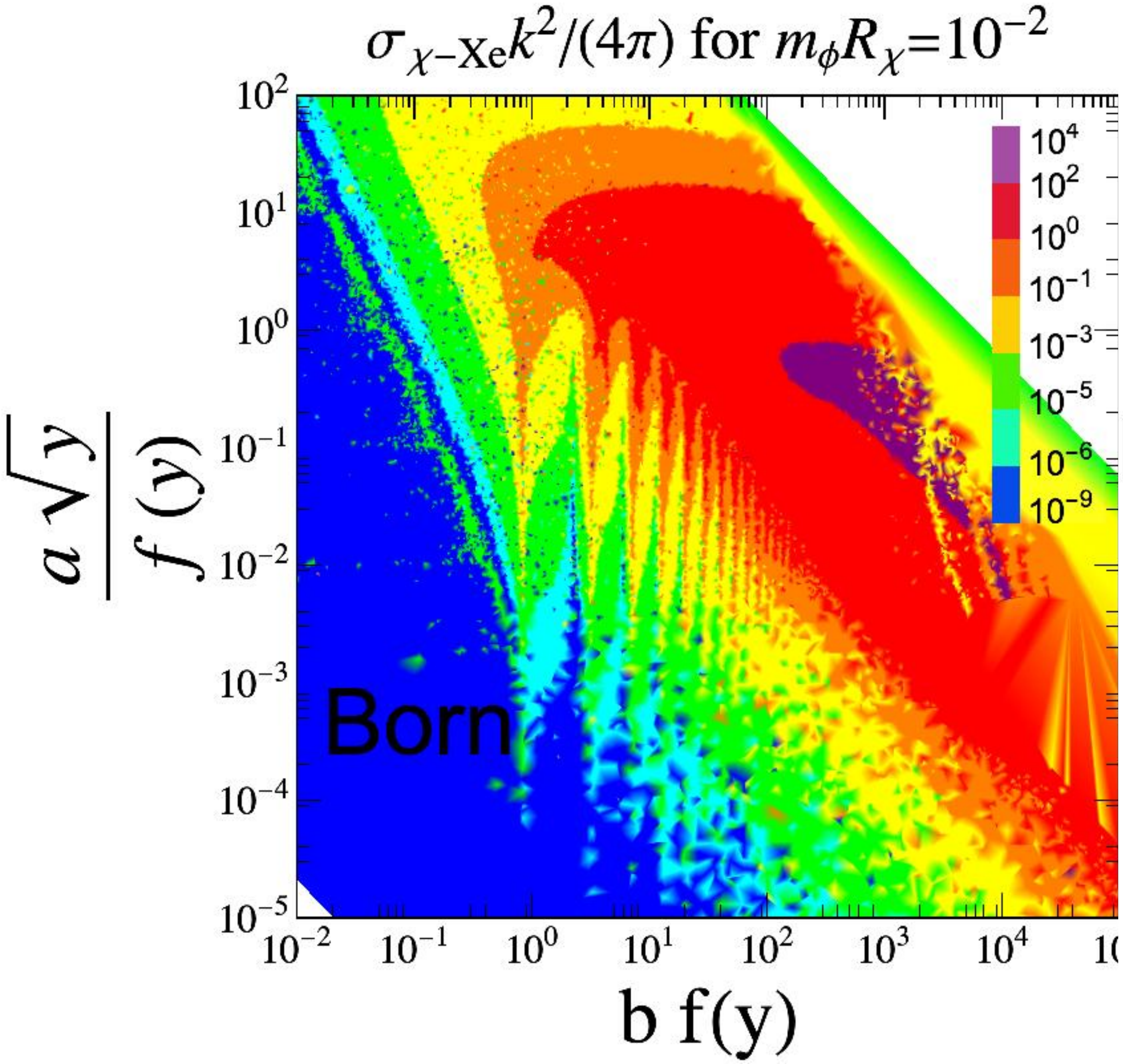
Puffy dark matter-proton scattering

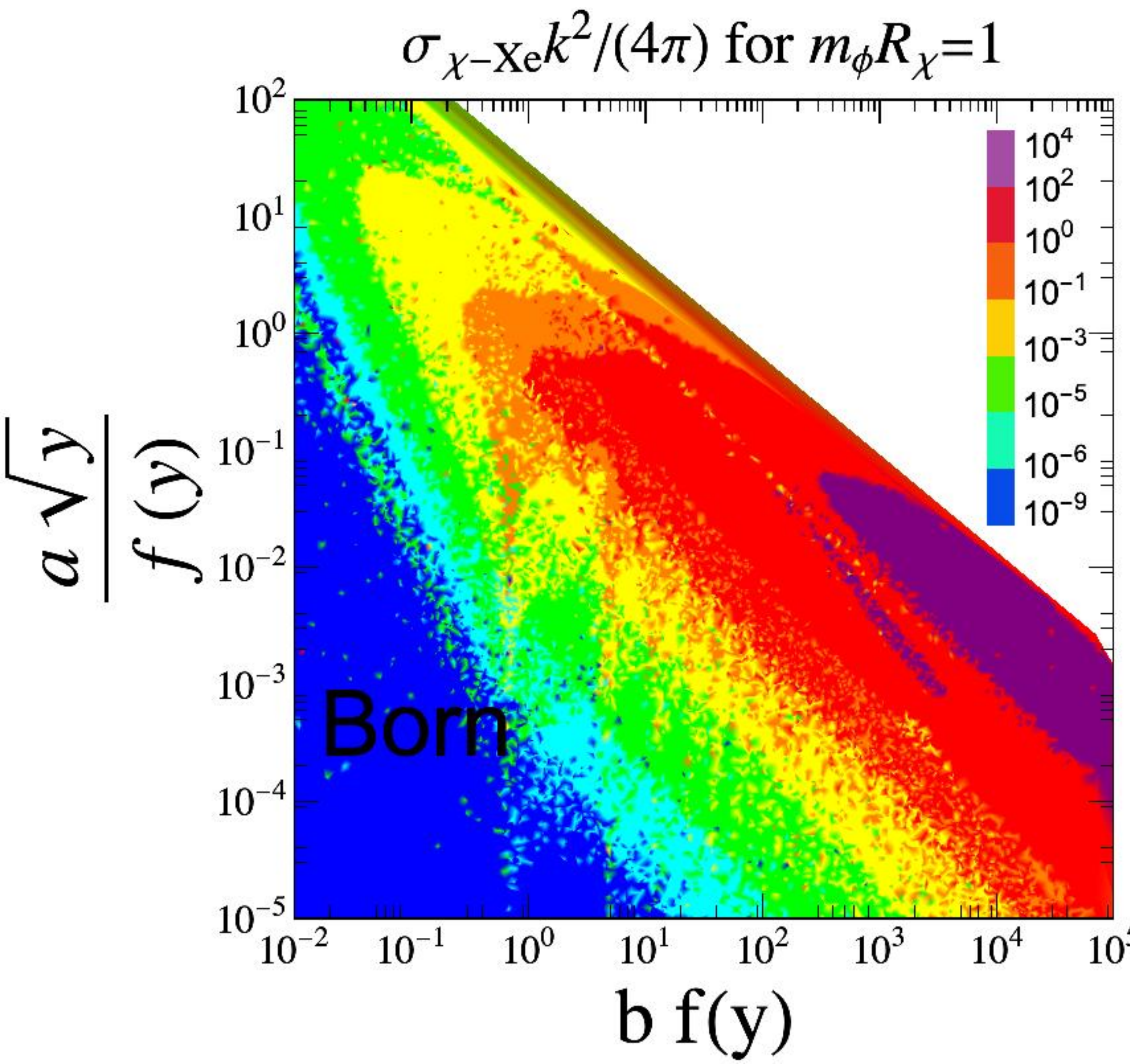




The scattering cross section between puffy dark matter and the nucleus can still be categorized into Born, resonance, and classical regions.

Puffy dark matter-xenon nucleus scattering





Compared to the case with nucleon number equal to 1, for the same y values the resonance peaks have larger values of the vertical parameter $a\sqrt{y}/f(y)$.

Dark matter has a internal structure

Detection of nugget-type dark matter

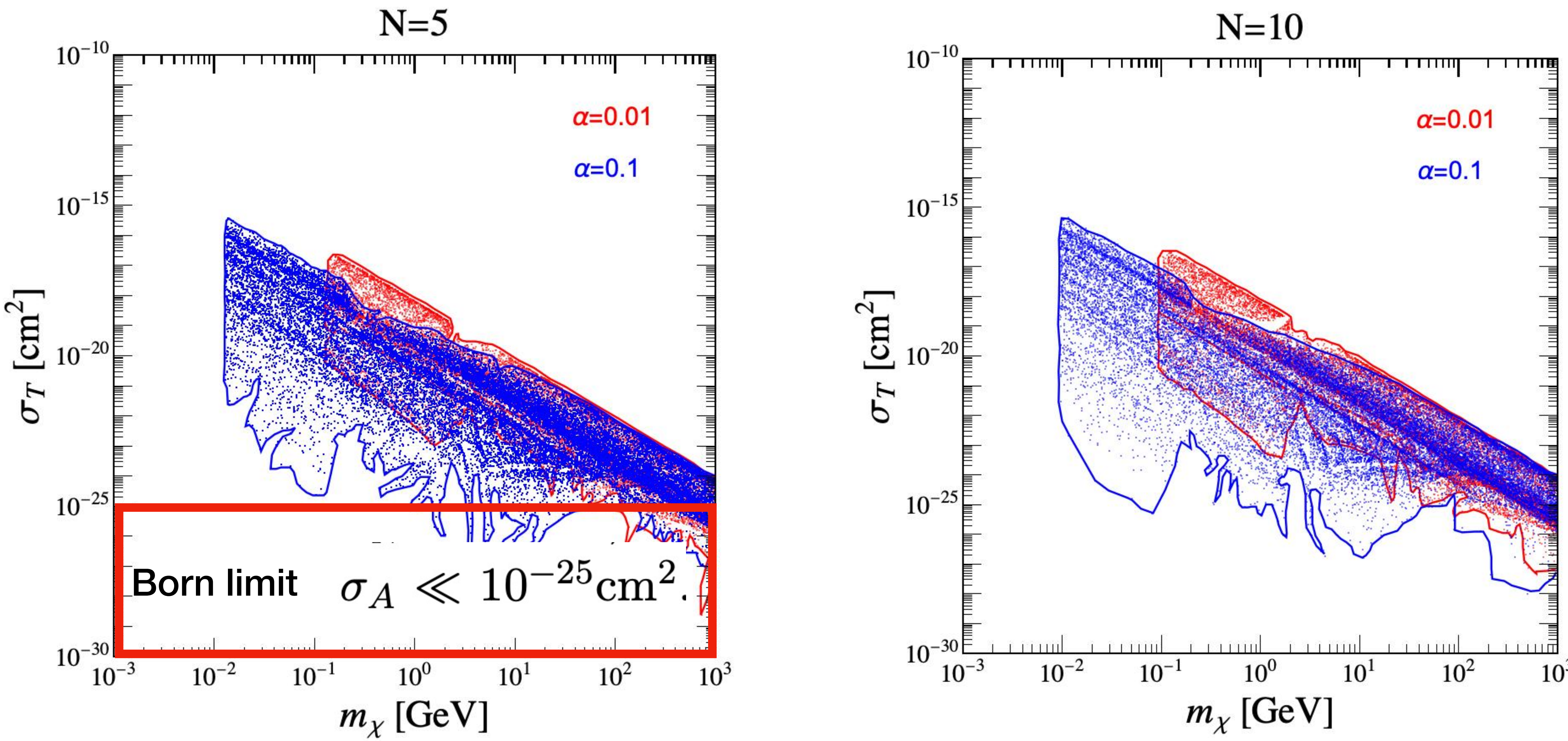
For a fixed α_ϕ and m_ϕ, there can be as many as three distinct regimes for the nugget profiles depending on N:

i) Small N: The nugget density is small and the constituents are largely non-relativistic. The density for the mediator remains relatively small, and the effective mass remains close to m_X. The nugget is small enough that $R \lesssim m_\phi^{-1}$, and the effect of the mediator mass, m_ϕ, is insignificant. Using the non-relativistic formula for a fermi gas, and assuming a Coulomb-like potential, a behavior $R \simeq \sqrt[3]{81\pi^2/(4N g_{\rm dof}^2 \alpha_\phi^3 m_X^3)}$ can be derived (see [6] for details).

ii) Medium N: The nugget is small enough, $R \lesssim m_\phi^{-1}$, that m_ϕ is largely unimportant, but the mediator density is large enough that m_* is significantly different from m_X. The constituents become relativistic, leading to large fermi presssure that extends the nugget sizes. A scaling $R \sim N^{2/3}$ can be obtained (see text).

iii) Large N (Saturation): R becomes much larger than m_ϕ^{-1}. The binding energy, mediator density and m_* all approach a constant. The nugget reaches a geometric limit where $R \sim N^{1/3}$.

Detection of nugget-type dark matter



For $\alpha = 0.01(0.1)$, the stability condition of the nugget-type bound state requires the dark matter particle mass to be greater than 0.1(0.01) GeV.

CONCLUSION

- Realistic Scattering of Puffy dark matter direct detection

1. The finite size effects of particles in dark matter direct detection cannot be neglected. Finite size effects of dark matter needs deep studies.
2. The scattering between particles of unequal sizes still allows for the classification into Born, resonant, and classical regimes..
3. Due to the stability condition of the dark matter bound-state, the viable dark matter-nucleus scattering cross section region is specified. This may be key difference to know a specific size dark matter model.

Thanks