Key points in constructing natural SUSY EFTs

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An 11-year research program addressing the naturalness problem of low-energy supersymmetry (a mechanism to protect scalar mass from large radiative corections) in the post-LHC era.

The work focuses on extensions of the MSSM, with the goal of constructing a theory that maintains fine-tuning at the percent level even in the face of prospective experimental constraints.

Key points in constructing feasible SUSY EFTs

1. Incorporating a mechanism to generate μ-prameter

SUSY EFT: Low-energy effective theory with manifest supersymmetry valid below the SUSY breaking scale:

- **Basic elements:** Gauge group, Chiral superfields, EWSB, etc.
- **arXiv:1309.2984 by Howard Baer, et. al.:**
  A natural SUSY EFT must incorporate an appropriate mechanism to generate μ-parameter, while soft SUSY breaking parameters can be taken as theoretical inputs.
  Messenger between hidden and visible sectors:
  Gravity, Gauge, or Anomaly-mediation.
- **Supergravity: When incorporating gravitational effects, the EFT contains supergravity multiplet where the gravitino emerges as the superpartner of the graviton.**
  Gravitino: Light in GMSB, while heavy in mSUGRA.
- **Strong CP problem: A SUSY EFT may include axion superfield, depending on model construction.**

Key points in constructing feasible SUSY EFTs

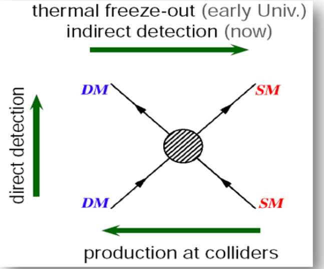
2. Employing inert DM to break the crossing symmetry between Ωh^2 and $\sigma_{\chi-N}$, which is predicted by simple WIMP DM theories.

Popular scenarios include **(totally six mechanisms)**

- **Secluded DM scenario:**
  Natural symmetry breaking through a **complex** framework;
- **Co-annihilation scenario:**
  Sometimes natural $\cdots$ **simple** $\cdots$;
- **Resonant scenario:**
  Usually unnatural, h**- and** Z**-funnels have been excluded**;
- **Leptophilic DM:**
  Becoming unnatural even through a **complex** framework;
- $\cdots$

Fermi-LAT, AMS...

XENON
PandaX
LUX...



LEP, Tevatron, LHC...

No new physics signal !

arXiv:2410.17036

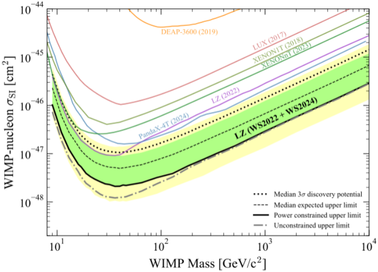


FIG. 5

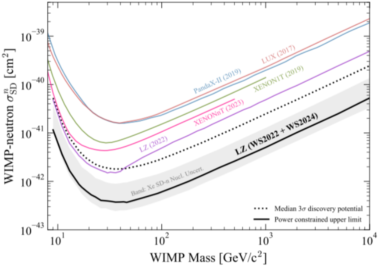


FIG. 6

Simple WIMP DM Theory:

DM mass: $m_{\rm DM} \sim 100$ GeV

Relic density: $\langle \sigma v \rangle \simeq 10^{-26} {\rm cm}^3 s^{-1}$

SI scattering: $\sigma^{\rm SI}_{\tilde{\chi}-N} \sim 10^{-45} {\rm cm}^2$,

SD scattering: $\sigma^{\rm SD}_{\tilde{\chi}-N} \sim 10^{-39} {\rm cm}^2$.

Examples-I: no secluded DM sector

Scenario: MSSM, $\tilde{\chi}_1^0 \lesssim 100$ GeV. **Ωh^2: $\tilde{\chi}_1^0$ should be Bino-like.**

Effective Couplings of DM:

$$C_{\tilde{B}\tilde{B}H_{\rm SM}} = g_1^2 \frac{v}{\mu}\left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right), \quad C_{\tilde{B}\tilde{B}H_{\rm NSM}} = g_1^2 \cos 2\beta \frac{v}{\mu},$$

$$-iC_{\tilde{B}\tilde{B}A_{\rm NSM}} = g_1^2 \frac{v}{\mu}\left(1 + \sin 2\beta \frac{m_{\tilde{\chi}_1^0}}{\mu}\right), \quad C_{\tilde{B}\tilde{B}Z} = \frac{g_1^3 \cos 2\beta}{2\sin\theta_W}\frac{v^2}{\mu^2}.$$

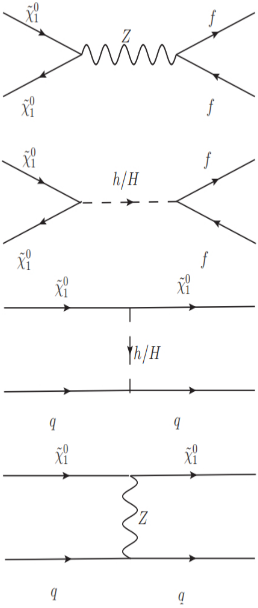
Formula for Direct Detection:

- Spin-independent scattering:

$$\sigma^{\rm SI}_{\tilde{B}-N} \simeq 5\times 10^{-45}{\rm cm}^2 \times \left[\left(\frac{C_{\tilde{B}\tilde{B}H_{\rm SM}}}{0.1}\right)\left(\frac{125\ {\rm GeV}}{m_h}\right)^2 - \cot 2\beta \left(\frac{C_{\tilde{B}\tilde{B}H_{\rm NSM}}}{0.1}\right)\left(\frac{125\ {\rm GeV}}{m_H}\right)^2\right]^2.$$

- Spin-dependent scattering:

$$\sigma^{\rm SD}_{\tilde{B}-N} \simeq C_N \times \left(\frac{C_{\tilde{B}\tilde{B}Z}}{0.1}\right)^2.$$



Phenomenology-I: No secluded DM sector in MSSM

Cases where SI LZ constraints are stronger than LHC constraints.

m_A	Negative $m_{\tilde{\chi}_1^0}/\mu$	Positive $m_{\tilde{\chi}_1^0}/\mu$
1TeV	$tan\beta \lesssim 9$	$tan\beta \lesssim 25$
2TeV	$tan\beta \lesssim 10$	$tan\beta \lesssim 27$

Experimental Constraints:

- Higgs data fit: $m_h \simeq 125$ GeV;
- LHC search for extra Higgs bosons;
- LHC search for supersymmetry;
- DM direct detection experiment LZ.

Preferred mass spectra:

- **LHC search for SUSY**: $\mu \gtrsim 900$ GeV;
- **LZ SD constraints**: $\mu \gtrsim 700$ GeV for $\tan\beta > 4$;
- **LZ SI constraints**: Sometimes stronger than LHC constraints.

Phenomenology-I: No secluded DM sector in MSSM

Two Types of Fine-Tuning:

1. **Z boson mass**
   - Mass formula: $m_Z^2 = \frac{2(m_{H_d}^2+\Sigma_d)-2(m_{H_u}^2+\Sigma_u)\tan^2\beta}{\tan^2\beta-1} - 2\mu^2$;
   - 0.8% fine-tuning without considering $\tilde{g}$ effect from RGE!
2. **DM annihilation**
   - The $1/\mu$ suppression of $C_{\tilde{B}\tilde{B}Z}$ and $C_{\tilde{B}\tilde{B}h}$ couplings requires m_χ to be precisely around $m_Z/2$ or $m_h/2$ for correct Ωh^2;
   - New fine-tuning!

Latest Conclusions:

1. **h and Z resonant annihilation mechanisms have been excluded by LZ-2024 results alone;**
   $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV from LHC search for compressed spectrum.
2. **Heavy Higgs resonant annihilation is strongly disfavored by LZ-2024 results:**

$$C_{\widetilde{B}\widetilde{B}H_{\rm SM}} = g_1^2 \frac{v}{\mu}\left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right) \sim \frac{g_1^2}{2}\frac{v}{\mu}, \quad C_{\widetilde{B}\widetilde{B}H_{\rm NSM}} = g_1^2 \frac{v}{\mu}.$$

Phenomenology-I: No secluded DM sector in MSSM

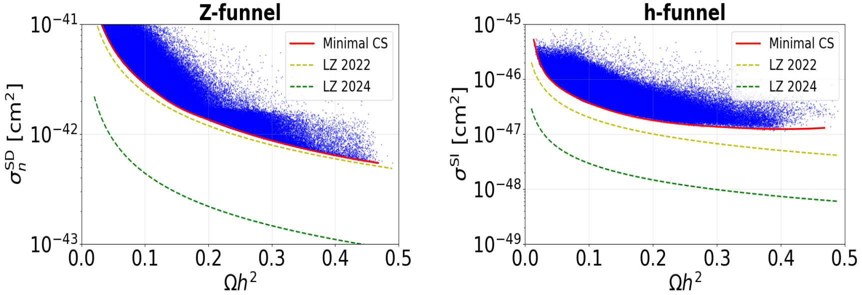


Figure: $\Omega h^2 - \sigma^{SD}_{\tilde{\chi}_1^0-N}$ **and** $\Omega h^2 - \sigma^{SI}_{\tilde{\chi}_1^0-N}$ **correlations for** Z**-funnel region (left panel) and** h**-funnel region (right panel). Radiative corrections to all involved quantities have been included.**

1. **Excessively large** μ **suppresses** $C_{\tilde{B}\tilde{B}Z}$ **and** $C_{\tilde{B}\tilde{B}h}$**, precluding the measured relic density even at resonance.**
2. **Conversely, the density requirement sets lower bounds on** $\sigma^{SD}_{\chi-N}$ **and** $\sigma^{SI}_{\chi-N}$ **at** Z**- and** h**-funnel regions, respectively.**

Phenomenology-II: Secluded DM sector in GNMSSM

- **Chiral Superfields**

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- **Superpotential — no ad hoc symmetry!**

$$W_{\mathrm{GNMSSM}} = W_{\mathrm{Y}} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3 + \mu \hat{H}_u \cdot \hat{H}_d + \frac{1}{2}\mu' \hat{S}^2 + \xi \hat{S}$$

◇ Solving domain wall and tadpole problems in Z_3-NMSSM.

◇ Z_3-violating terms originate from unified theories with a Z_4^n or Z_8^n sym..

◇ The $\xi \hat{S}$ term can be eliminated by field magentaefinitions.

GNMSSM: DM Mass and Couplings

Singlino-dominated DM

$\mu_{eff} \equiv \frac{\lambda}{\sqrt{2}} v_s$, $\mu_{tot} \equiv \mu + \mu_{eff}$.

- Neutralino mass matrix in basis $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\right)$:

$$m_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu_{\mathbf{tot}} & -\frac{1}{\sqrt{2}} v_u \lambda \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu_{\mathbf{tot}} & 0 & -\frac{1}{\sqrt{2}} v_d \lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}} v_u \lambda & -\frac{1}{\sqrt{2}} v_d \lambda & \mathbf{m_N} \end{pmatrix}$$

Mass and couplings of the singlino-dominated DM are given by:

$$\mathbf{m_{\tilde{\chi}_1^0}} \simeq \mathbf{m_N} + \frac{\mathbf{1}}{\mathbf{2}} \frac{\lambda^{\mathbf{2}} \mathbf{v^2} (\mathbf{m_{\tilde{\chi}_1^0}} - \mu_{\mathbf{tot}} \sin \mathbf{2}\beta)}{\mathbf{m^2_{\tilde{\chi}_1^0}} - \mu^{\mathbf{2}}_{\mathbf{tot}}} \simeq \mathbf{m_N}, \quad \mathbf{m_N} \equiv \sqrt{\mathbf{2}} \kappa \mathbf{v_s} + \mu',$$

Theoretical features at the cost of model economy:

- ◇ **All particle masses are independent:** m_N, $\mu_{\rm tot}$, m_{h_s}, m_{A_s}, m_h, $\cdots$;
- ◇ **All couplings are independent:** λ, κ, g_1, g_2, g_s;
- ◇ **Singlet-related parameters are scarcely limited:** $m_{\tilde{\chi}_1^0}/m_N$, m_{h_s}, m_{A_s}, κ. **$\tilde{S}$-dominated DM: $|m_N| \lesssim \mu_{\rm tot}, |M_1|, |M_2|$;**

In contrast, Z_3-NMSSM: $2|\kappa| \lesssim \lambda$ to predict $\tilde{S}$-dominated DM.

Phenomenology-II: Secluded DM sector in GNMSSM

Scenario: GMSSM, $\tilde{\chi}_1^0 \lesssim 100$ GeV. $\tilde{\chi}_1^0$**: Bino- or Singlino-like**.

Key characteristics of DM physics:

- **For Bino-like DM**, they are same as those of the MSSM;
- **For Singlino-like DM**, $\tilde{S}$, h_s and A_s may form a **secluded** DM sector, specifically
  κ-controlled annihilations: $\tilde{S}\tilde{S} \to h_s A_s, h_s h_s, A_s A_s$;
  λ-controlled DM-nucleon scatterings: $\sigma^{\rm SI}_{\chi-N} \propto \lambda^2\kappa^2$, $\sigma^{\rm SD}_{\chi-N} \propto \lambda^4$.
  Consequently, the light DM scenario easily aligns with experimental results.

Phenomenology-II: Secluded DM sector in GNMSSM

Other involved couplings: relying on $\mu_{\rm tot}$, λ, and κ.

$$C_{\tilde{B}\tilde{B}Re[S]} = iC_{\tilde{B}\tilde{B}Im[S]} = -\frac{\lambda g_1^2 \sin 2\beta}{2}\frac{v^2}{\mu_{\rm tot}^2},$$

$$C_{\tilde{B}\tilde{S}H_{\rm SM}} = -2\sqrt{2}\lambda g_1 \cos 2\beta \frac{v}{\mu_{\rm tot}}, \quad iC_{\tilde{B}\tilde{S}A_{\rm NSM}} = -\sqrt{2}\lambda g_1 \cos 2\beta \frac{(M_1 + M_{\tilde{S}})\, v}{\mu_{\rm tot}^2},$$

$$C_{\tilde{B}\tilde{S}H_{\rm NSM}} = \sqrt{2}\lambda g_1 \frac{v}{\mu_{\rm tot}}\left[2\sin 2\beta + \frac{(M_1 - M_{\tilde{S}})}{\mu_{\rm tot}}\right],$$

$$C_{\tilde{B}\tilde{S}Re[S]} = iC_{\tilde{B}\tilde{S}Im[S]} = \sqrt{2}\lambda^2 g_1 \cos 2\beta \frac{v^2}{\mu_{\rm tot}^2}, \quad C_{\tilde{B}\tilde{S}Z} = \frac{\sqrt{2}\lambda g_1^2 \sin 2\beta}{\sin\theta_W}\frac{v^2}{\mu_{\rm tot}^2},$$

$$C_{\tilde{S}\tilde{S}H_{\rm SM}} = -2\lambda^2 \sin 2\beta \frac{v}{\mu_{\rm tot}} + 2\lambda^2 \frac{m_{\tilde{S}} v}{\mu_{\rm tot}^2}, \quad C_{\tilde{S}\tilde{S}H_{\rm NSM}} = -2\lambda^2 \cos 2\beta \frac{v}{\mu_{\rm tot}},$$

$$C_{\tilde{S}\tilde{S}Re[S]} = iC_{\tilde{S}\tilde{S}Im[S]} = \lambda^3 \sin 2\beta \frac{v^2}{\mu_{\rm tot}^2} - 2\kappa\left(1 - \lambda^2 \frac{v^2}{\mu_{\rm tot}^2}\right),$$

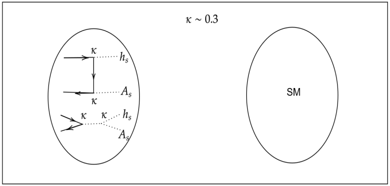
$$iC_{\tilde{S}\tilde{S}A_{\rm NSM}} = 2\lambda^2 \frac{v}{\mu_{tot}}\left[1 - \sin 2\beta \frac{m_{\tilde{S}}}{\mu_{tot}}\right] \quad C_{\tilde{S}\tilde{S}Z} = -\frac{\lambda^2 g_1 \cos 2\beta}{\sin\theta_W}\frac{v^2}{\mu_{\rm tot}^2}.$$

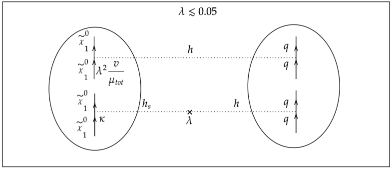
Formula for Direct Detections

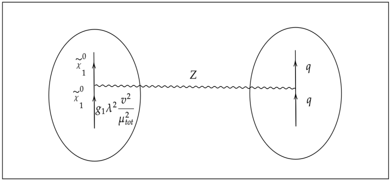
- Spin-independent scattering:

$$\sigma^{\rm SI}_{\tilde{\chi}-N} \simeq 5\times 10^{-45}{\rm cm}^2 \times \left[\left(\frac{125\ {\rm GeV}}{m_h}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}h}}{0.1}\right) + \left(\frac{125\ {\rm GeV}}{m_H}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}H}}{0.1}\right) + \left(\frac{125\ {\rm GeV}}{m_{h_s}}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}h_s}}{0.1}\right)\right]^2.$$

- Spin-dependent scattering: $\sigma^{\rm SD}_{\tilde{\chi}-N} \simeq C_N \times \left(\frac{C_{\tilde{\chi}\tilde{\chi}Z}}{0.1}\right)^2$.







Phenomenology-II: Secluded DM sector in GNMSSM

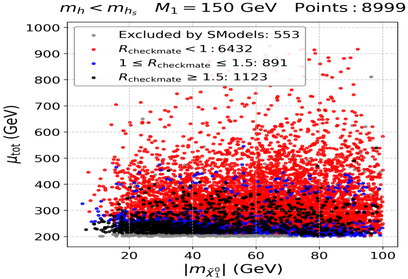


Figure: **Samples, surviving all experimental constraints except those from LHC, are projected onto $m_{\tilde{\chi}_1^0} - \mu_{\rm tot}$ plane.**

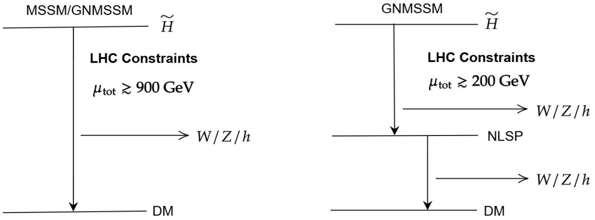


Figure: **Different cases of LHC constraints.**

Note Added:

- **SUSY DM is not always WIMP; popular non-WIMP candidates include gravitino and axion-axino.**
- **Key points 2 can not be applied to $\tilde{G}$ and $a/\tilde{a}$.**
- **Gravitino DM $\tilde{G}$:** $g_{\tilde{G}} \sim m_{\rm soft}/(m_{\tilde{G}} M_{\rm Pl})$
  - ✓ **Couplings with ordinary matter are suppressed by $M_{\rm Pl}$ for $1~{\rm KeV} \lesssim m_{\tilde{G}} \lesssim 1~{\rm MeV} \Rightarrow$ Hard to directly detect!**
  - ✓ **Collider signatures dominated by long-lived NLSPs $\Rightarrow 10^{-9}s \lesssim \tau \lesssim 10^{-6}s$ relevant for FASER;**
  - ✓ **UV freeze in mechanism $\Rightarrow$ Strong constraints to acquire measured relic density: $T_R \sim (10 - 10^4)$ GeV;**
  - ✓ **For other mass range, strongly constrained by cosmology!**
- **WIMP: strongly constrained by DD experiments;**
  $\tilde{G}$: usually involving non-standard cosomology,
  e.g., predicting the low T_R needs fine-tuning:
  $Y_{\rm Inflator} \sim 10^{-11}$, significantly weaker than gravity!
- **$a/\tilde{a}$: Different story from $\tilde{G}$ given that their couplings only depend on f_a, although some properties are similar.**

Details of $\tilde{G}$ DM candidate:

- Produced through **non-standard early-Universe mechanisms**: **non-thermal production, scatterings in the thermal plasma, and delayed decays of heavier particles**.
- Highly model dependent DM mass. However, relic density, BBN, small-scale structure, and reheating constraints often select **specific mass ranges**, sometimes favoring **light DM**.
- **Strongly constrained by conventional cosmology**, including Ωh^2, BBN, T_R, $\tau_{\rm NLSP}$, and structure formation.
- DM itself usually **does not** affect the production rates or the early stages of collider cascade decays. Instead, **a long-lived NLSP may alter the collider-signature topology** from **conventional prompt missing-E_T events** to **displaced vertices, delayed decays, disappearing tracks, or other long-lived-particle signatures**.

Conclusion: MSSM

MSSM is disfavored by naturalness argument!

- Owing to intrinsic structure in its theoretical framework, **the fine-tuning of MSSM** is at 0.01% level, i.e., $\Delta_{FT}^{\rm BG} \sim 10000$;
  - ◇ Barbieri-Giudice Measurement:
  $$\Delta_{FT}^{\rm BG} \equiv max\left(\frac{\partial \ln O}{\partial \ln p_i}\right)$$
  O: a physics observable, e.g., m_Z, Ωh^2, etc.;
  p_i: ith input parameter of the theory.
  - ◇ Expected $\Delta_{FT}^{\rm BG}$ during the LEP era: $\Delta_{FT}^{\rm BG} \lesssim 10$;
  - ◇ Acceptable $\Delta_{FT}^{\rm BG}$ in the post-LHC era: $\Delta_{FT}^{\rm BG} \lesssim 300$.
- **Only** constraints from Ωh^2 measurement and LZ experiment **have excluded** Z- and h-resonant annihilation mechanisms;
- **Additionally**, LHC searches for compressed spectrum set a lower bound on DM mass, $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV.

Conclusion: Beyond MSSM

Unexpected theoretical advantages in gauge extensions;
B-L extended NMSSM: an economic extension that can alleviate FT problem of MSSM to $\mathcal{O}(1\%)$ even when future ..

1. **Economic extension of MSSM** - No dimensional parameters in its superpotential;
2. **Elegant solution to μ-problem** - **Free from domain-wall problem with a minimal framework**;
3. **Automatically explaining the conservation of R-parity;**
4. **Dynamically generating neutrino mass and mixing;**
5. **Singlino-modulated and sequestered DM** - **Naturally suppressed DM-nucleon scattering**;
6. **Well-motivated DM annihilation** - **BLino - Higgsino coannihilation**;
7. **Simple phenomenology** - **Similar to MSSM, with Higgsino acting as DM candidate.** $\cdots$

Conclusion: WIMP

Recent advancements in WIMP DM theory:

- **Keep the idea of freeze-out and thermal relic;**
- **Introduce more complex structure to break crossing symmetry between Ωh^2 and $\sigma_{\chi-N}$;**
- **Phenomenology significantly differs from traditional MSSM prediction because of gauge-singlet nature of DM, which leads to lengthened decay chain of sparticles.**

E.g., $m_{\tilde{H}} \simeq 200$ GeV is experimentally allowed!

No hints of DM and sparticles in experiments are consistent with this picture!

Examples

1. **Secluded DM Sector**

   Singlino-dominated DM in GNMSSM:

   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{4\{W/Z/h\}} E_T^{\mathrm{Miss}}$$

   τ-flavored righthanded sneutrino DM in seesaw extension:

   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{2\tau 2\{W/Z/h\}} E_T^{\mathrm{Miss}}$$

Conclusion: WIMP

Example

2. **Co-annihilation**
   - **Most natural co-annihilation: same gauge multiplet;**
     - ✓ Higgsino $SU(2)_L$ doublet: $m_{\tilde{\chi}_1^0} \simeq 1.1$ TeV for measured Ωh^2;
     - ✓ Wino $SU(2)_L$ triplet: $m_{\tilde{\chi}_1^0} \simeq 3.0$ TeV for measured Ωh^2, **excluded by DM indirect search experiments**.
   - **Another natural co-annihilation: Higgsino-partnered. BLino-Higgsino co-annihilation in B-L NMSSM is well-motivated, and yields measured Ωh^2 for any mass value.**

Phenomenology of B-L NMSSM:

Similar to MSSM with **Higgsino** acting **as DM candidate**, except for the subtlety about Higgsino signal:

(1) Missing momentum; **(2) Displaced vertex;**
(3) Missing track; **(4) "Orphan" signal,**

which **depends** on the Higgsino mass spectrum and lifetime, and also on LHC search strategies.

thanks!