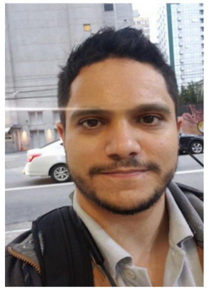
Atomic Effects
in
Dark Matter Scattering



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SFG, Pedro Pasquini, Jie Sheng [JHEP 05 (2022) 088]

SFG, Jie Sheng, Chuan-Yang Xing [arXiv:2509.14534]

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SHANGHAI JIAO TONG UNIVERSITY

紫金山暗物质研讨会 @ 烟台

October 18, 2025

李政道研究所
Tsung-Dao Lee Institute

Outline

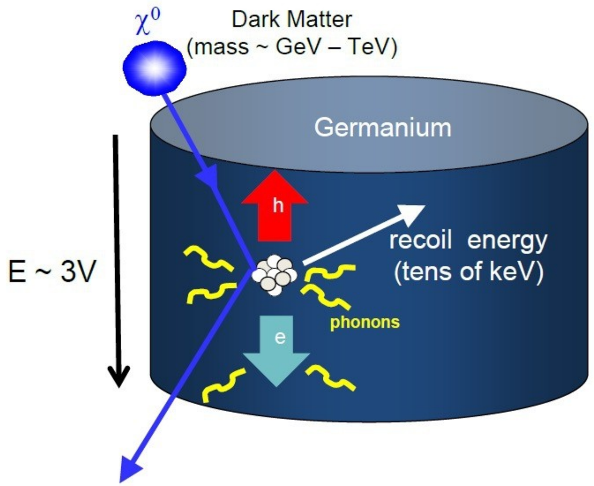
1) Brief Overview

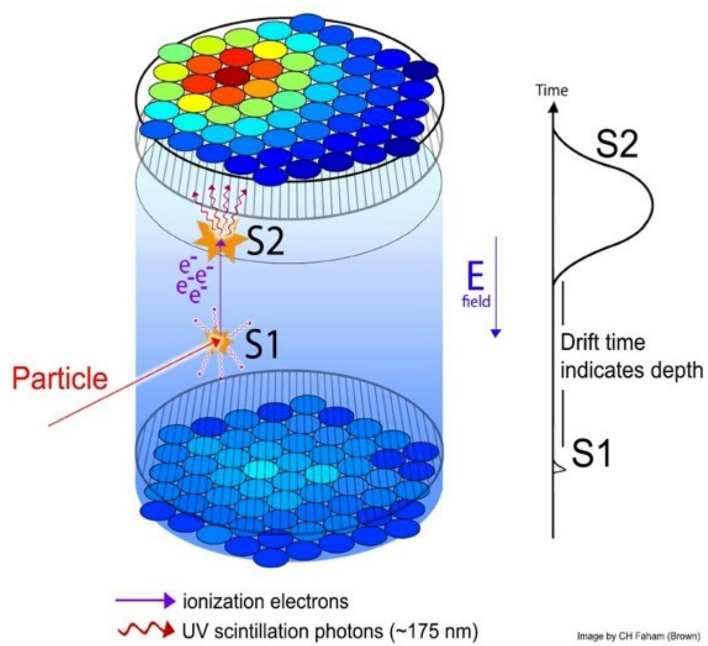
2) Atomic Effects with 2nd Quantization

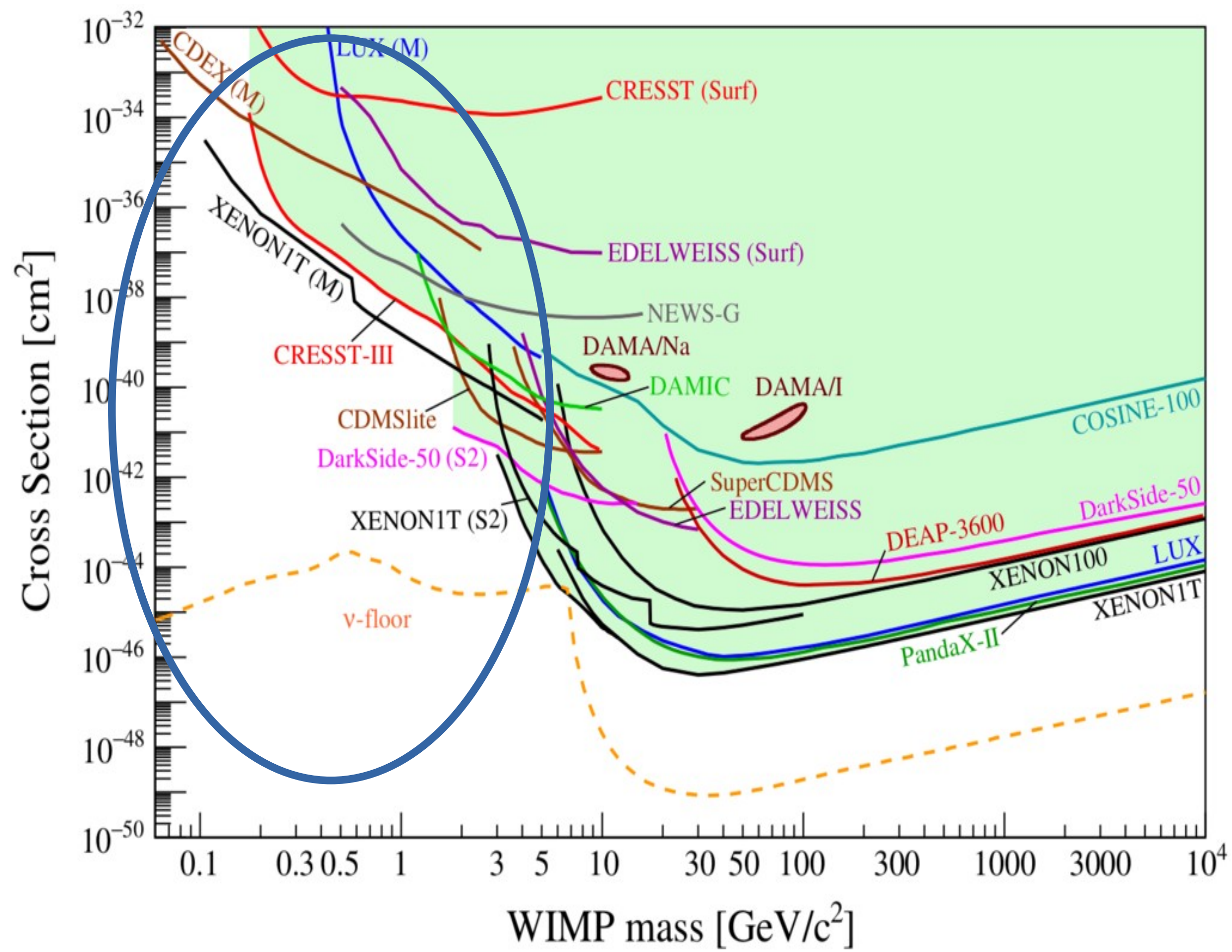
3) Relativistic Effects

4) Summary

Threshold & DM-Electron Scattering







APPEC Committee Report [2104.07634]

Overcoming Threshold

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- **Elastic Scattering**

$$\chi + N/e \to \chi + N/e \qquad E_r \approx \frac{4 m_\chi m_N}{(m_\chi + m_N)^2} T_\chi$$

- **Exothermic**

$$X' e^- \to X e^- \qquad E_R \simeq \Delta m \left(1 - \frac{v_{\rm DM}}{v_e} \cos\theta_e\right)$$

He, Wang & Zheng [JCAP21, 2007.04963]

Aboubrahim, Althueser, Klasen, Nath & Weinheimer [2207.08621]

- **Fermionic absorption**

$$\chi + e \to \nu + e \qquad E_r \approx \frac{m_\chi^2}{2 m_e} \qquad m_\chi \sim \mathcal{O}(10)\,\mathrm{keV}$$

Dror, Elor, McGehee & Yu [2011.01940]

See also Dror, Elor & McGehee [1905.12635, 1908.10861]

Absorption DM vs Neutrino

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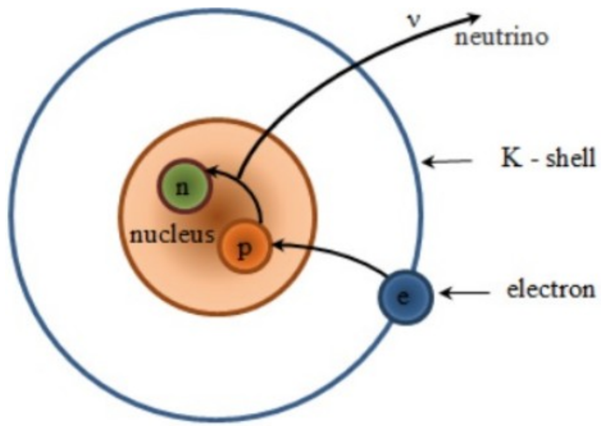
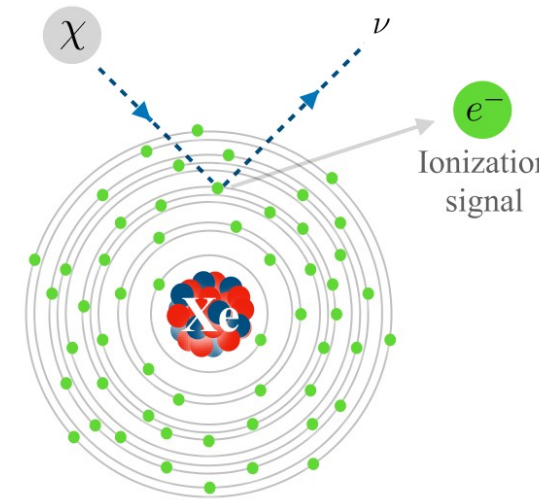
王淦昌

1942 – Kan Chang Wang proposed using K-shell electron capture for detecting neutrino





$$e^- + {}^{Z}_{A}N \rightarrow {}^{Z-1}_{A}N + \nu$$

$$\chi + e \rightarrow \nu + e$$

Kan Chang Wang, *A Suggestion on the Detection of the Neutrino*, **Phys. Rev., 61, 97 (1942)**

Kan Chang Wang, *Proposed Methods of Detecting the Neutrino* , **Phys. Rev., 71, 645-646 (1947)**

Absorption DM

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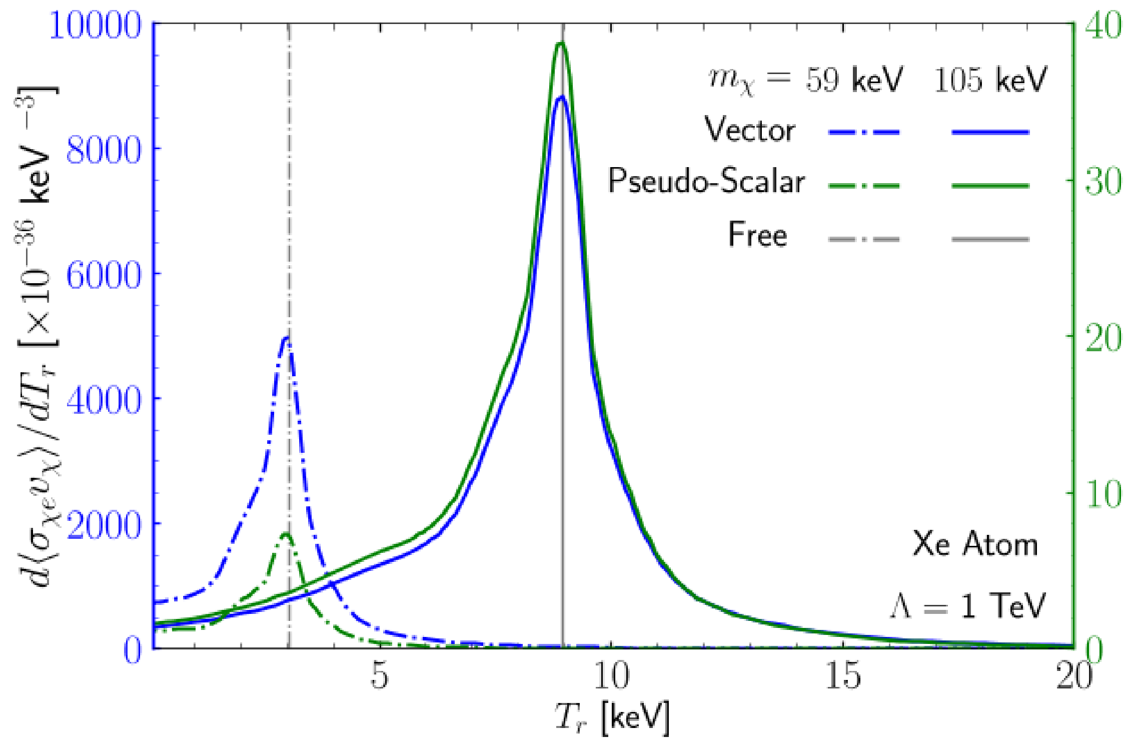
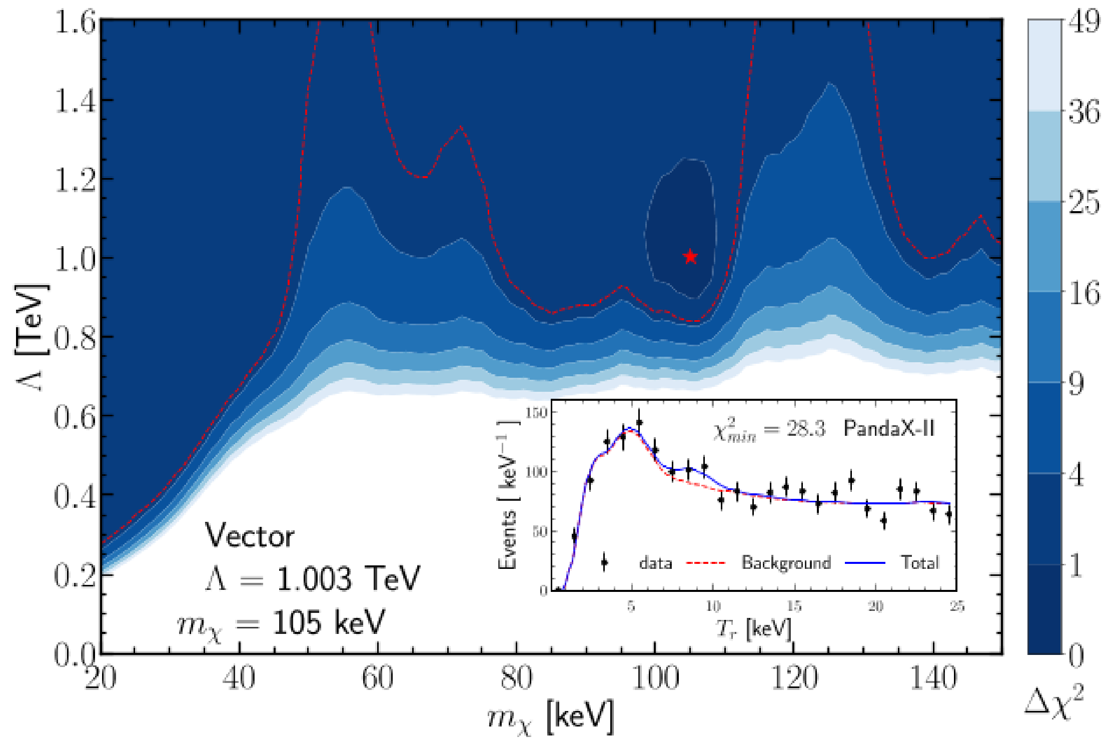
$$E_r = \frac{m_\chi^2}{2(m_e + m_\chi)} \approx \frac{m_\chi^2}{2m_e}$$





$$\frac{d\langle\sigma_{\chi e} v_\chi\rangle}{dT_r} = \sum_{nl} (4l+2) \frac{1}{T_r} \frac{m_\chi - \Delta E_{nl}}{16\pi m_e^2 m_\chi} |\mathcal{M}|^2(\mathbf{q}) K_{nl}(T_r, |\mathbf{q}|),$$

SFG, Xiao-Gang He, Xiao-Dong Ma, Jie Sheng [JHEP 05 (2022) 191]

SFG, Pedro Pasquini, Jie Sheng [JHEP 05 (2022) 088]

K-Factor

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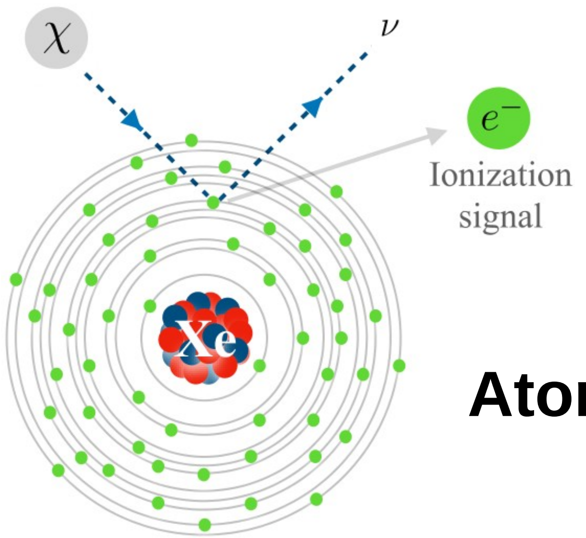
$$\sigma v_{1\to 2} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} \int \frac{d^3 q}{4\pi} \delta\left(\Delta E_{1\to 2} + \frac{q^2}{2m_\chi} - \vec{q}\cdot\vec{v}\right) \times |F_{\rm DM}(q)|^2 |f_{1\to 2}(\vec{q})|^2$$



$$\overline{\sigma}_e \equiv \frac{\mu_{\chi e}^2 \overline{|\mathcal{M}_{\rm free}(\alpha m_e)|^2}}{16\pi m_\chi^2 m_e^2} \qquad |\mathcal{M}|^2 \equiv |\mathcal{M}_{\rm free}|^2 \times |f_{i\to f}(\boldsymbol{q})|^2$$

Atomic Form Factor $$f_{1\to 2}(\vec{q}) = \int d^3 x\, \psi_2^*(\vec{x}) \psi_1(\vec{x}) e^{i\vec{q}\cdot\vec{x}}$$

DM Form Factor $$F_{\rm DM}(q) = \frac{m_{A'}^2 + \alpha^2 m_e^2}{m_{A'}^2 + q^2} \simeq \begin{cases} 1\,, & m_{A'} \gg \alpha m_e \\ \frac{\alpha^2 m_e^2}{q^2}\,, & m_{A'} \ll \alpha m_e \end{cases}$$

Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu [JHEP 05 (2016) 046]

Assumption!: Particle & atomic parts can be factorized

Outline

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Cross Section with Bound States

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Cross section = Probability x Area

$$\sigma \equiv \int P(\boldsymbol{b}) d^2 \boldsymbol{b}$$

The mass dimension of σ is inherited from the area.

Quantum transition probability

$$P \equiv \left| {}_{\text{out}} \langle \phi_\chi \phi_e | \phi_\chi \phi_e \rangle_{\text{in}} \right|^2$$

$$|\phi_\chi \phi_e\rangle_{\text{in}} \to \int \frac{d^3 \boldsymbol{p}_\chi}{(2\pi)^3} \frac{\phi_\chi(\boldsymbol{p}_\chi) e^{-i\boldsymbol{b}\cdot\boldsymbol{p}_\chi}}{\sqrt{2E_{\boldsymbol{p}_\chi}}} |i; \boldsymbol{p}_\chi\rangle_{\text{in}} \qquad |\phi_\chi \phi_e\rangle_{\text{out}} \to |\boldsymbol{p}'_\chi; f\rangle$$

Unfortunately, initial & final states are not plane waves.

Cross Section with Non-Plane Wave State

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$$\mathcal{T} \equiv \langle f; \boldsymbol{p}'_\chi | iT | i; \boldsymbol{p}_\chi \rangle$$

$$\sigma \equiv \int d^2\boldsymbol{b} \int d\Omega'_e \frac{d^3\boldsymbol{p}'_\chi}{(2\pi)^3 2E_{\boldsymbol{p}'_\chi}} \left| \int \frac{d^3\boldsymbol{p}_\chi}{(2\pi)^3 \sqrt{2E_{\boldsymbol{p}_\chi}}} e^{i\boldsymbol{p}_\chi \cdot \boldsymbol{b}} \phi_\chi(\boldsymbol{p}_\chi) \; \mathcal{T}(\boldsymbol{q}) \right|^2$$

$$\mathcal{T}(\boldsymbol{q}) \equiv \mathcal{M}(\boldsymbol{q})(2\pi)\delta\left(\Delta E - E_{\boldsymbol{p}_\chi} + E_{\boldsymbol{p}'_\chi}\right)$$

$$\sigma^{\rm ion}_{\rm bound} = \frac{1}{2|\boldsymbol{v}_\chi| E_{\boldsymbol{p}_\chi}} \int d\Omega'_e \int \frac{d^3\boldsymbol{p}'_\chi}{(2\pi)^3 2E_{\boldsymbol{p}'_\chi}} \overline{|\mathcal{M}(\boldsymbol{p}_\chi, \boldsymbol{p}'_\chi)|^2} (2\pi)\delta(E_{\boldsymbol{p}'_\chi} - E_{\boldsymbol{p}_\chi} + \Delta E)$$

2^{nd} Quantized Bound & Ionized Fields

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$$e_B(x) \equiv \sum_{n\kappa m_j} a_{n\kappa m_j} e^{-iE_{n\kappa}t} \psi_{n\kappa m_j}(\boldsymbol{x})$$

$$\int \psi^\dagger_{n\kappa m_j}(\boldsymbol{x}) \psi_{n'\kappa' m'_j}(\boldsymbol{x}) d^3\boldsymbol{x} = \delta_{nn'}\delta_{\kappa\kappa'}\delta_{m_j m'_j}$$

$$[\psi_{n\kappa m_j}(\boldsymbol{x})] = 3/2$$

$$[a_{n\kappa m_j}] = [a^\dagger_{n\kappa m_j}] = 0$$

$$\left\{ a_{n\kappa m_j}, a^\dagger_{n'\kappa' m'_j} \right\} = \delta_{nn'}\delta_{\kappa\kappa'}\delta_{m_j m'_j}$$

$$\langle n'\kappa' m'_j | n\kappa m_j \rangle = \delta_{nn'}\delta_{\kappa\kappa'}\delta_{m_j m'_j}$$

$$e_I(x) \equiv \sum_{\kappa m_j} \int \frac{dT_r}{2\pi} a_{T_r\kappa m_j} \psi_{T_r\kappa m_j}(\boldsymbol{x}) e^{-iT_r t}$$

$$\int \psi^\dagger_{T_r\kappa m_j}(\boldsymbol{x}) \psi_{T'_r\kappa' m'_j}(\boldsymbol{x}) d^3\boldsymbol{x} = 2\pi \delta_{\kappa\kappa'}\delta_{m_j m'_j}\delta(T_r - T'_r)$$

$$[\psi_{T_r\kappa m_j}(\boldsymbol{x})] = 1$$

$$[a_{T_r\kappa m_j}] = -1/2$$

$$\{ a_{T_r\kappa m_j}, a^\dagger_{T'_r\kappa' m'_j} \} = 2\pi \delta_{\kappa\kappa'}\delta_{m_j m'_j}\delta(T_r - T'_r)$$

$$\langle T'_r\kappa' m'_j | T_r\kappa m_j \rangle = 2\pi \delta_{\kappa\kappa'}\delta_{m_j m'_j}\delta(T_r - T'_r)$$

Final-State Phase Space

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Ionized state does not contain the direction information!

$$e_I(x) \equiv \sum_{\kappa m_j} \int \frac{dT_r}{2\pi} a_{T_r \kappa m_j} \psi_{T_r \kappa m_j}(\boldsymbol{x}) e^{-iT_r t} \quad \Rightarrow \quad d\Omega'_e = \sum_{\kappa m_j} \int \frac{dT_r}{2\pi}$$

How to justify such phase space?

Jacobian $\quad \delta^{(3)}(\boldsymbol{p}) = \frac{1}{|\boldsymbol{p}|^2} \delta(|\boldsymbol{p}|) \delta(\varphi) \delta(\cos\theta)$

$$\delta^{(3)}(\boldsymbol{p})\, d^3\boldsymbol{p} = \delta(|\boldsymbol{p}|) d|\boldsymbol{p}| = \delta(T_r) dT_r$$

$$\sigma^{ion}_{n\kappa} = \frac{1}{2|\boldsymbol{v}_\chi| E_{\boldsymbol{p}_\chi}} \sum_{\kappa' m'_j m_j} \int \frac{dT_r}{2\pi} \int \frac{d^3 \boldsymbol{p}'_\chi}{(2\pi)^3 2E_{\boldsymbol{p}'_\chi}} \overline{|\mathcal{M}(\boldsymbol{p}_\chi, \boldsymbol{p}'_\chi)|^2} (2\pi) \delta(E_{\boldsymbol{p}'_\chi} - E_{\boldsymbol{p}_\chi} + \Delta E)$$

Factorization

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Full field for an atomic electron:

$$e(x) = \sum_{n\kappa m_j} a_{n\kappa m_j} \psi_{n\kappa m_j}(\boldsymbol{x}) e^{-iE_{n\kappa}t} + \sum_{\kappa m_j} \int \frac{dT_r}{2\pi} a_{T_r\kappa m_j} \psi_{T_r\kappa m_j}(\boldsymbol{x}) e^{-iT_r t}$$

Bound state　　　　Ionized state

Follow QFT language to obtain scattering matrix element

$$\mathcal{M}(\boldsymbol{p}_\chi, \boldsymbol{p}'_\chi) \equiv \bar{u}_\chi(\boldsymbol{p}'_\chi)\Gamma_A u_\chi(\boldsymbol{p}_\chi) D_{AB}(q) \int d^3\boldsymbol{y} e^{i\boldsymbol{q}\cdot\boldsymbol{y}} \bar{\psi}_f(\boldsymbol{y})\Gamma_B \psi_i(\boldsymbol{y})$$

$$\psi_{a,m_s}(\boldsymbol{x}) \approx \frac{1}{\sqrt{2}} \begin{pmatrix} (1 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \\ (1 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \end{pmatrix} \qquad f_{a,m_s}(\boldsymbol{r}) \equiv \phi_a(\boldsymbol{r})\eta_{m_s}$$

$$\int d^3\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \bar{\psi}_f(\boldsymbol{r})\Gamma_B\psi_i(\boldsymbol{r}) = \frac{1}{2m_e} \bar{u}_{s'}(m_e)\Gamma_B u_s(m_e) \int d^3\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \phi^*_f(\boldsymbol{r})\phi_i(\boldsymbol{r})$$

Factorization with Electron at Rest

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Factorization is possible, but only with electron at rest!

$$\mathcal{M} \equiv \frac{1}{2m_e} \mathcal{M}_0(\boldsymbol{q}) \times \int d^3\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \phi^*_{E_r l' m_l'}(\boldsymbol{r}) \phi_{nlm_l}(\boldsymbol{r})$$

$$\mathcal{M}_0 \equiv \bar{u}_\chi(\boldsymbol{p}'_\chi)\Gamma_A u_\chi(\boldsymbol{p}_\chi) D_{AB}(q) \bar{u}(m_e)\Gamma_B u(m_e)$$

Comparing with the one in literature:

$$|\mathcal{M}|^2 \equiv |\mathcal{M}_{\text{free}}|^2 \times |f_{i\to f}(\boldsymbol{q})|^2 \quad \Rightarrow \quad |\mathcal{M}|^2 \equiv |\mathcal{M}_0(m_e)|^2 \times |f_{i\to f}(\boldsymbol{q})|^2$$

Factorizable only in the non-relativistic limit.

Application to Absorption DM

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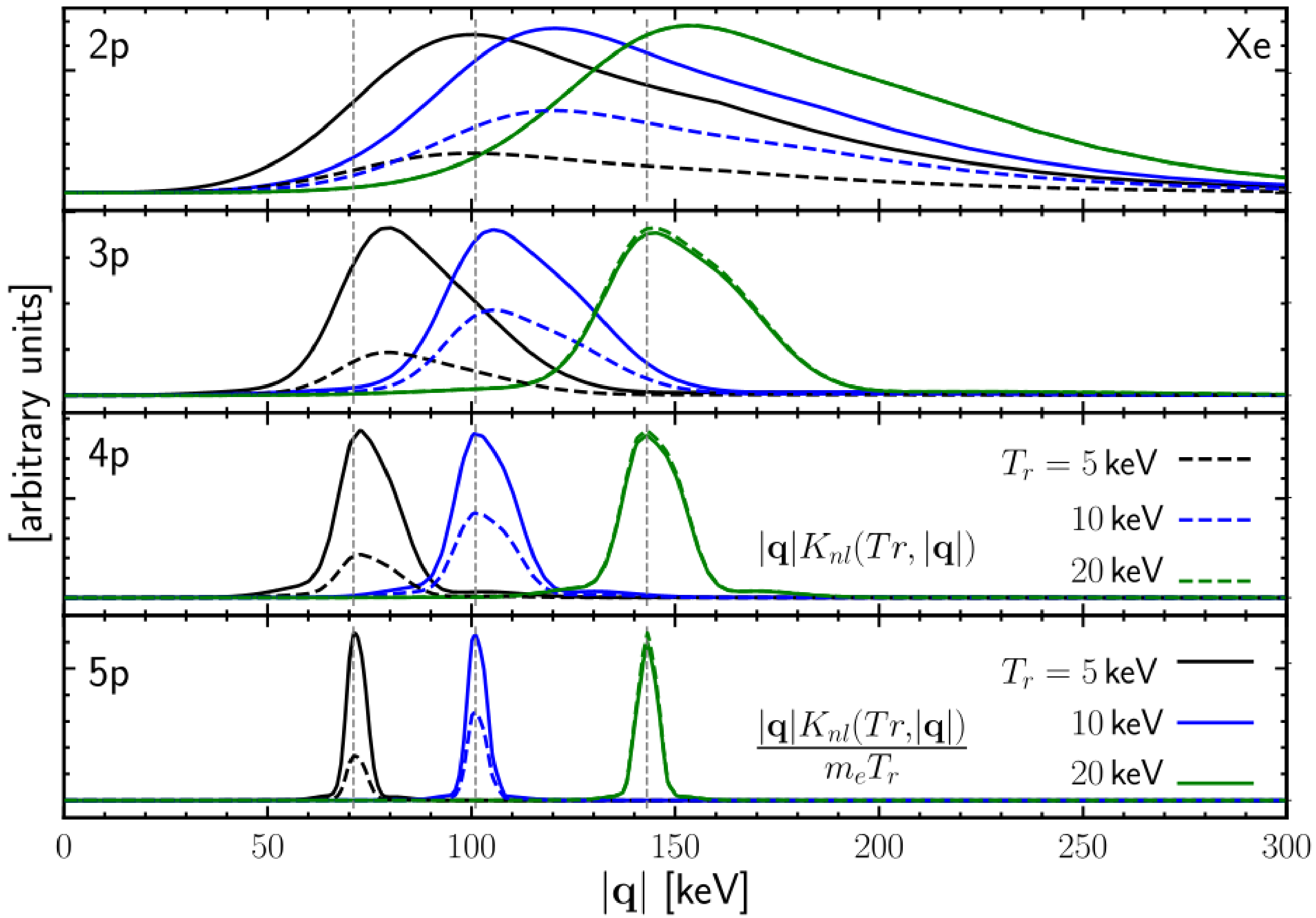
$$\frac{d\langle \sigma_{\chi e} v_\chi \rangle}{dT_r} = \sum_{nl} (4l+2) \frac{1}{T_r} \frac{m_\chi - \Delta E_{nl}}{16\pi m_e^2 m_\chi} |\mathcal{M}|^2(\mathbf{q}) \boxed{K_{nl}(T_r, |\mathbf{q}|)},$$



SFG, Pedro Pasquini, Jie Sheng [JHEP 05 (2022) 088]

Negative Cross Section

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Dark photon mediator

$$\mathcal{L} \supset g_\chi \bar{\chi} \gamma^\mu \chi A'_\mu + g_e \bar{e} \gamma^\mu e A'_\mu + \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu$$

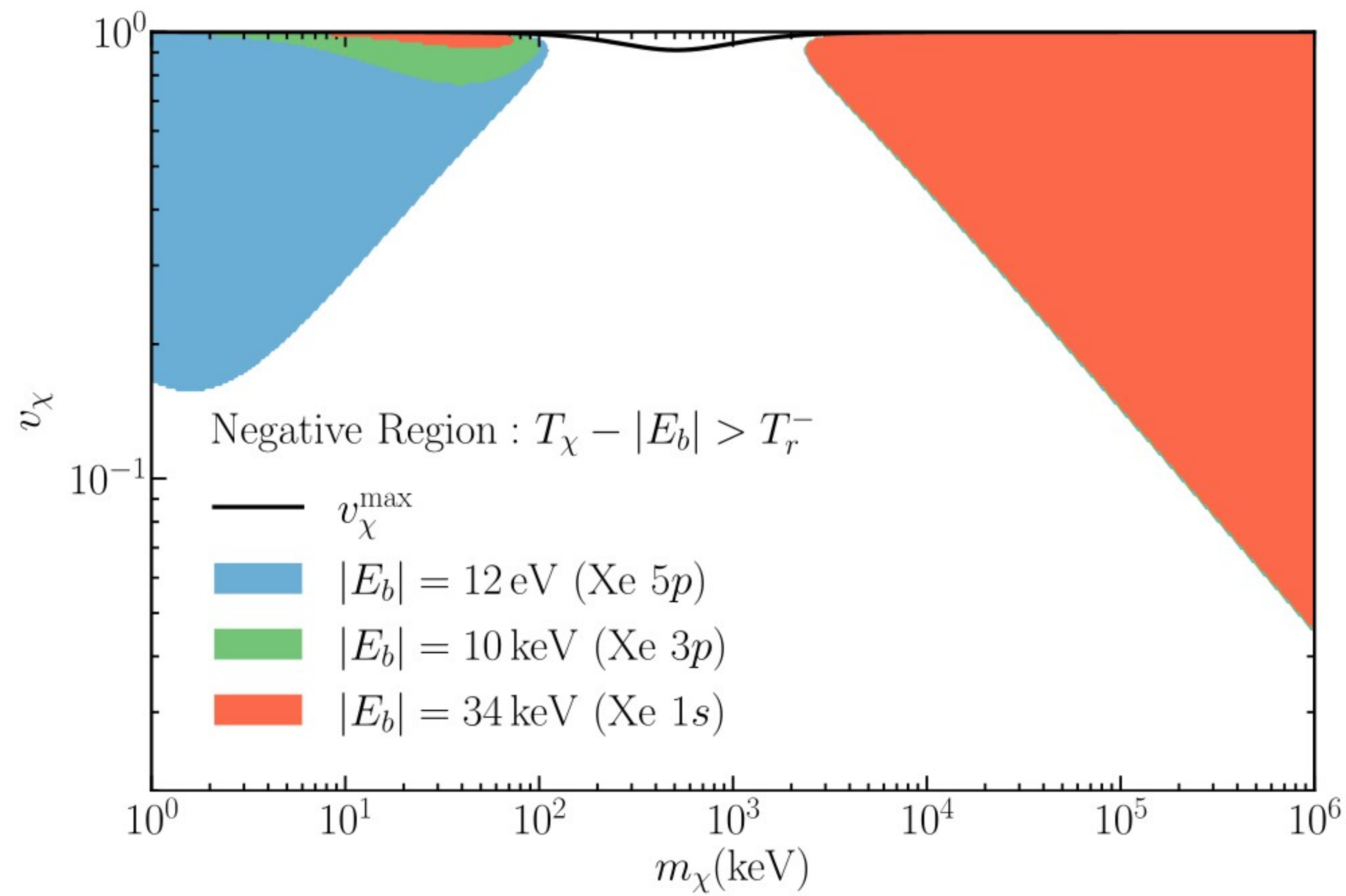
$$\overline{|\mathcal{M}_{\rm free}|^2} = \frac{32 m_\chi [m_e T_r^2 - T_r(m_e^2 + m_\chi^2 + 2 m_e m_\chi + 2 m_e T_\chi) + 2 m_e (m_\chi + T_\chi)^2]}{(t - m_{A'}^2)^2}$$

$$T_\chi < \frac{(m_e + m_\chi)^2 - 2\sqrt{2} m_e m_\chi}{2(\sqrt{2} - 1) m_e}$$

Potential problem for cosmic ray boosted DM (CRDM)!

$\mathcal{M}_{\rm free}$ **X**

Bound election is no longer on shell!



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Why we need relativistic effect?

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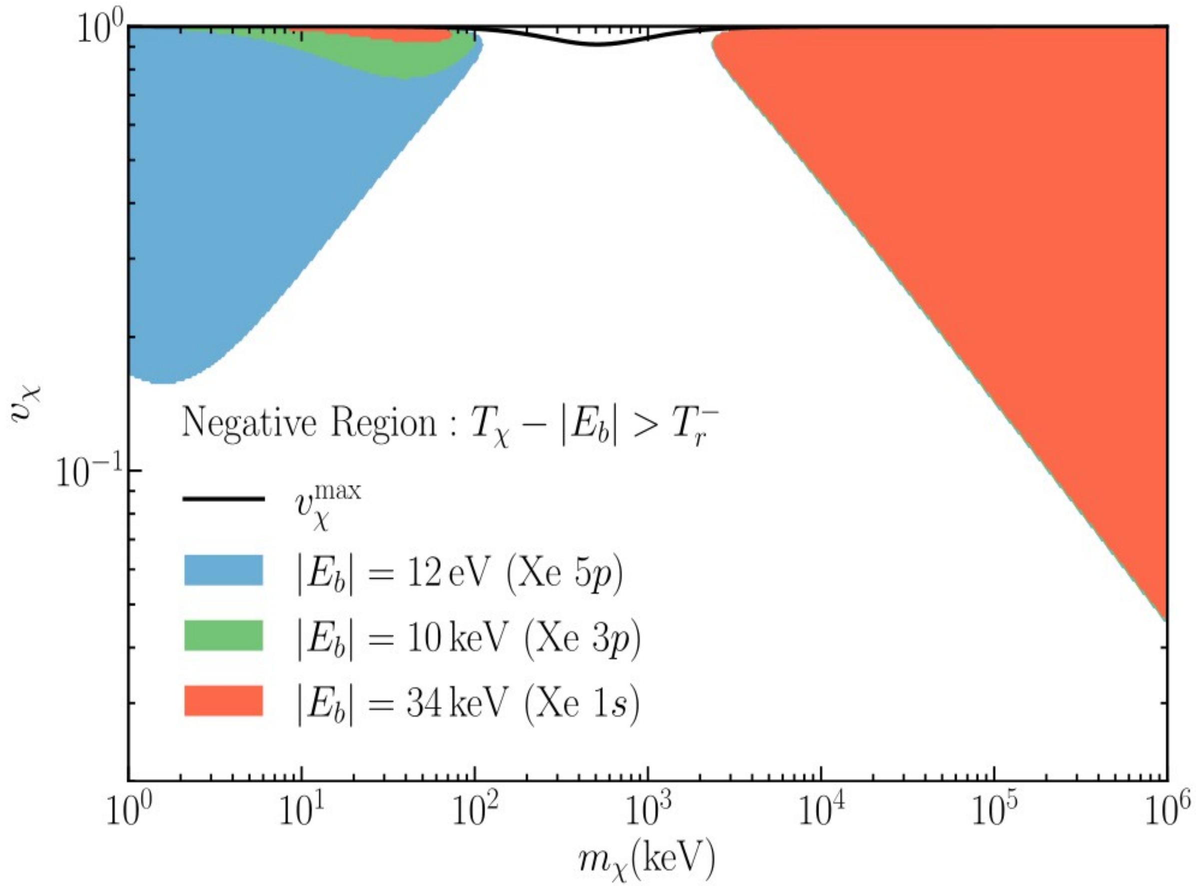

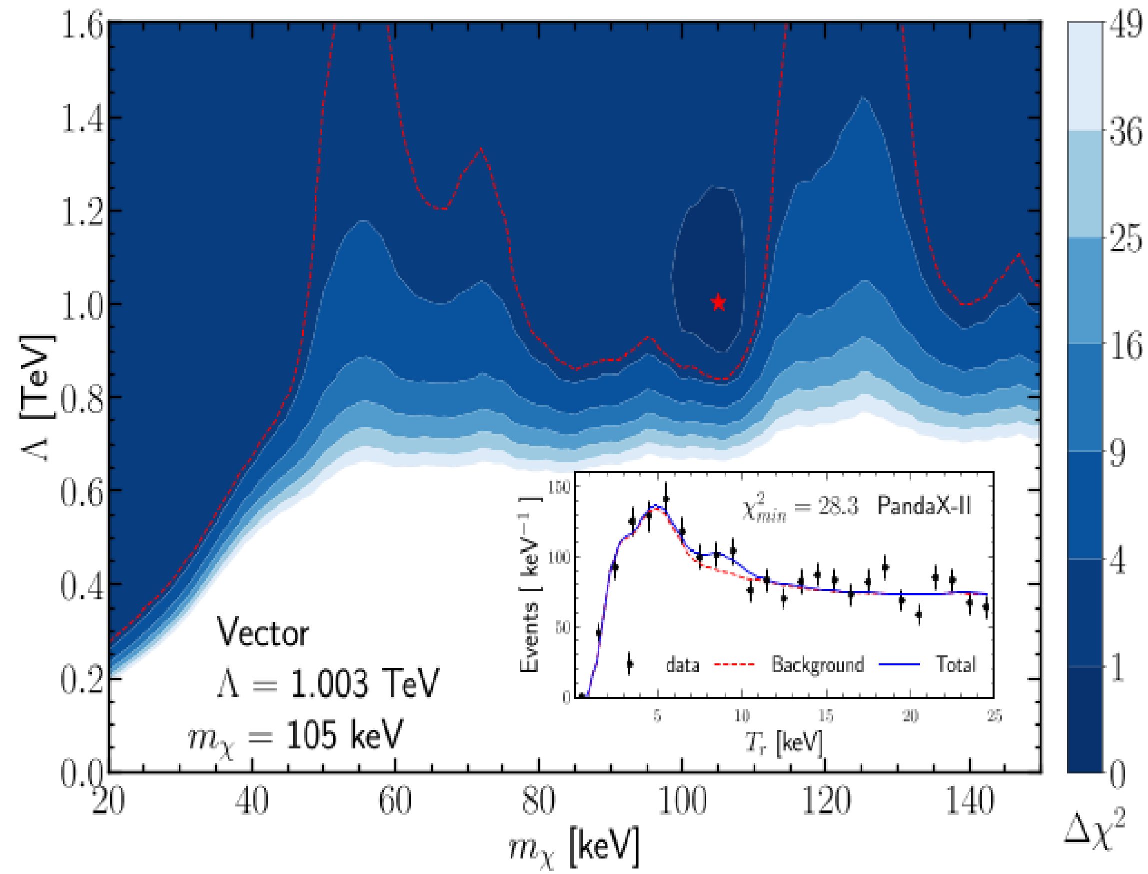

1) CRDM

2) Absorption DM

Non-Factorization Except Scalar Type

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Factorization is no longer possible!

$$\overline{|\mathcal{M}(\boldsymbol{q})|^2} = \frac{1}{J_\chi} \mathrm{Tr}[(\not{p}_\chi + m_\chi)\Gamma_A(\not{p}'_\chi + m_{\chi'})\Gamma'_A] |D_{AB}(\boldsymbol{q})|^2$$

$$\times \mathrm{Tr}\left[\int d^3\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \bar{\psi}_f(\boldsymbol{r}) \Gamma_B \psi_i(\boldsymbol{r}) \int d^3\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \bar{\psi}_i(\boldsymbol{r}) \Gamma'_B \psi_f(\boldsymbol{r})\right]$$

For easier comparison, adopt scalar interaction form

$$\frac{d\sigma^{S,P}_{n\kappa} |\boldsymbol{v}_\chi|}{dT_r} = \frac{2|\kappa|}{16\pi^2} \int_{\boldsymbol{q}_{\min}}^{\boldsymbol{q}_{\max}} \frac{|\boldsymbol{q}| d|\boldsymbol{q}|}{|\boldsymbol{p}_\chi| E_{\boldsymbol{p}_\chi}} \overline{|M^{SS}_\chi|^2} D^2_{SS}(q) K^S_{n\kappa}(\boldsymbol{q}, \Delta E)$$

$$K^S_{n\kappa}(\boldsymbol{q}, \Delta E) \equiv \frac{1}{2|\kappa|} \sum_{\kappa' m'_j m_j} \left| \int d^3\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \psi^\dagger_{T_r \kappa' m'_j}(\boldsymbol{r}) \gamma^0 \psi_{n\kappa m_j}(\boldsymbol{r}) \right|^2$$

Atomic form factor

Dirac Fermion Solution

$$\psi_{a,m_s}(\boldsymbol{x}) \approx \frac{1}{\sqrt{2}} \begin{pmatrix} (1 + i\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \\ (1 - i\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \end{pmatrix}$$

a = Good Quantum Numbers

Non-Relativistic $a \equiv (n l m_l)$

Relativistic $a \equiv (n \kappa m_j)$ **Spin-orbital coupling**

$$\psi_{n(T_r)\kappa m_j}(r, \Omega) = \frac{1}{r} \begin{pmatrix} a P_{n(T_r)\kappa}(r) \sqrt{\frac{l+\frac{1}{2}+a m_j}{2l+1}} Y_l^{m_j - 1/2} \\ P_{n(T_r)\kappa}(r) \sqrt{\frac{l+\frac{1}{2}-a m_j}{2l+1}} Y_l^{m_j + 1/2} \\ -a i Q_{n(T_r)\kappa}(r) \sqrt{\frac{l'+\frac{1}{2}-a m_j}{2l'+1}} Y_{l'}^{m_j - 1/2} \\ i Q_{n(T_r)\kappa}(r) \sqrt{\frac{l'+\frac{1}{2}+a m_j}{2l'+1}} Y_{l'}^{m_j + 1/2} \end{pmatrix}$$

Large Component

Small Component

$$j = |\kappa| - \frac{1}{2} \qquad l = j + \frac{1}{2}\mathrm{Sign}(\kappa) \qquad a \equiv -\mathrm{Sign}(\kappa)$$

Relativistic vs Non-Relativistic

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$$\psi_{a,m_s}(\boldsymbol{x}) \approx \frac{1}{\sqrt{2}} \begin{pmatrix} (1 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \\ (1 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}/2m_e) f_{a,m_s}(\boldsymbol{r}) \end{pmatrix}$$

a = Good Quantum Numbers

Non-Relativistic $a \equiv (n l m_l)$

Relativistic $a \equiv (n \kappa m_j)$

States	$3s$	$3p$		$3d$	
Non-Relativistic	$l=0$ $N_e = 2$	$l=1$ $N_e = 6$		$l=2$ $N_e = 10$	
Relativistic	$\kappa=-1$ $j=\frac{1}{2}$ $l=0$ $N_e=2$	$\kappa=1$ $j=\frac{1}{2}$ $l=1$ $N_e=2$	$\kappa=-2$ $j=\frac{3}{2}$ $l=1$ $N_e=4$	$\kappa=2$ $j=\frac{3}{2}$ $l=2$ $N_e=4$	$\kappa=-3$ $j=\frac{5}{2}$ $l=2$ $N_e=6$

$$j = |\kappa| - \frac{1}{2} \qquad l = j + \frac{1}{2}\mathrm{Sign}(\kappa) \qquad a \equiv -\mathrm{Sign}(\kappa)$$

Why we need relativistic effect?

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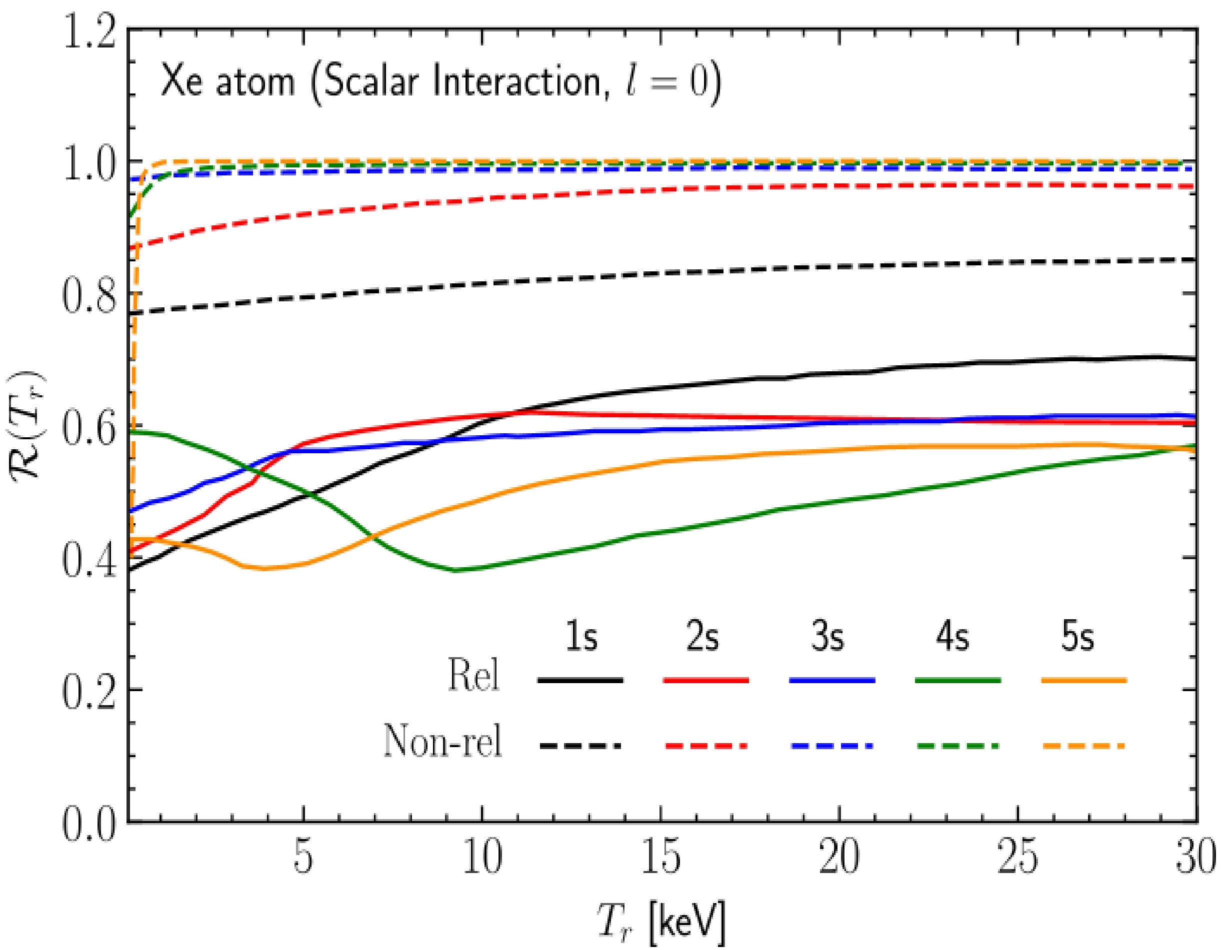

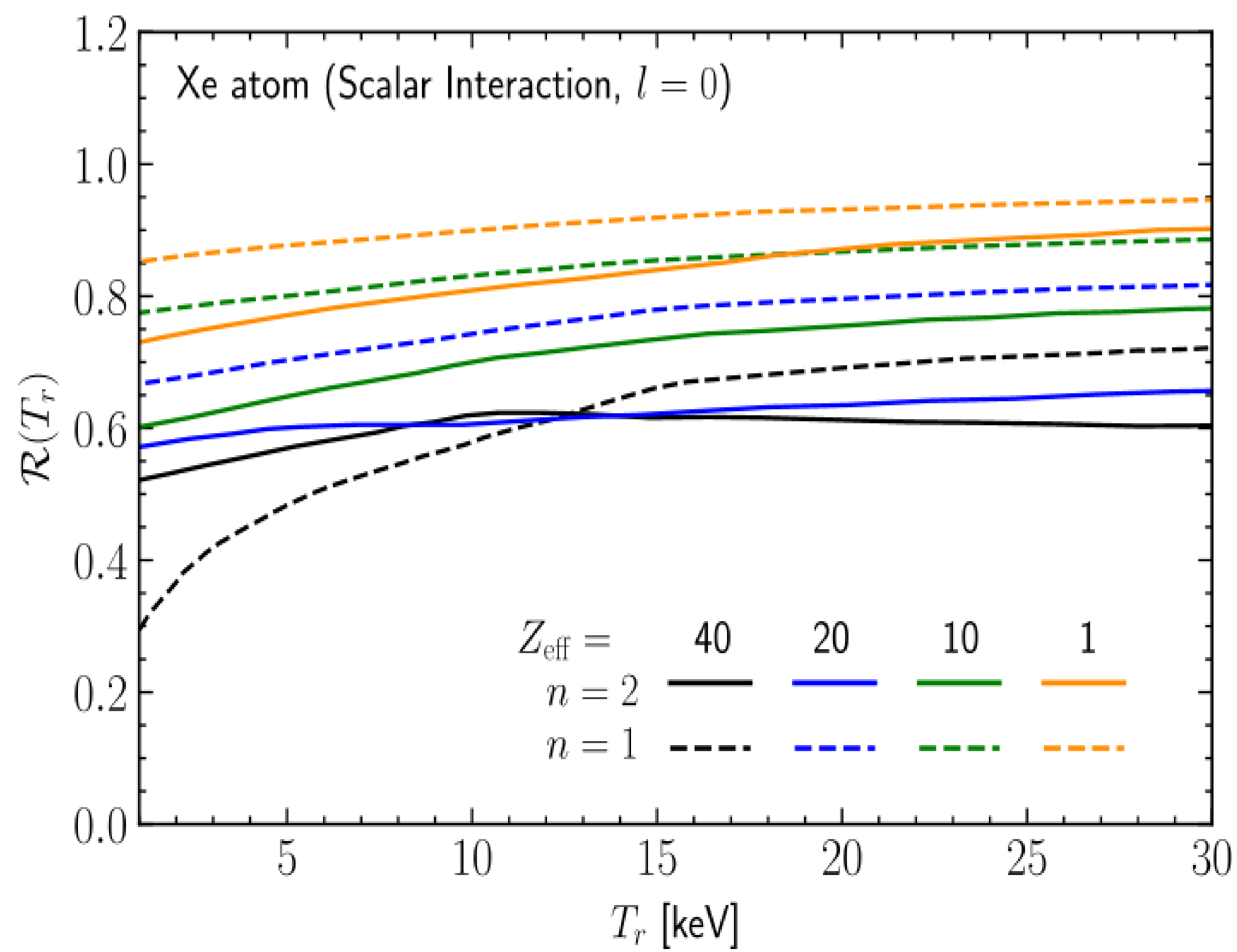

The relativistic effect can be as large as 50%!

Relativistic vs Non-Relavistic WF

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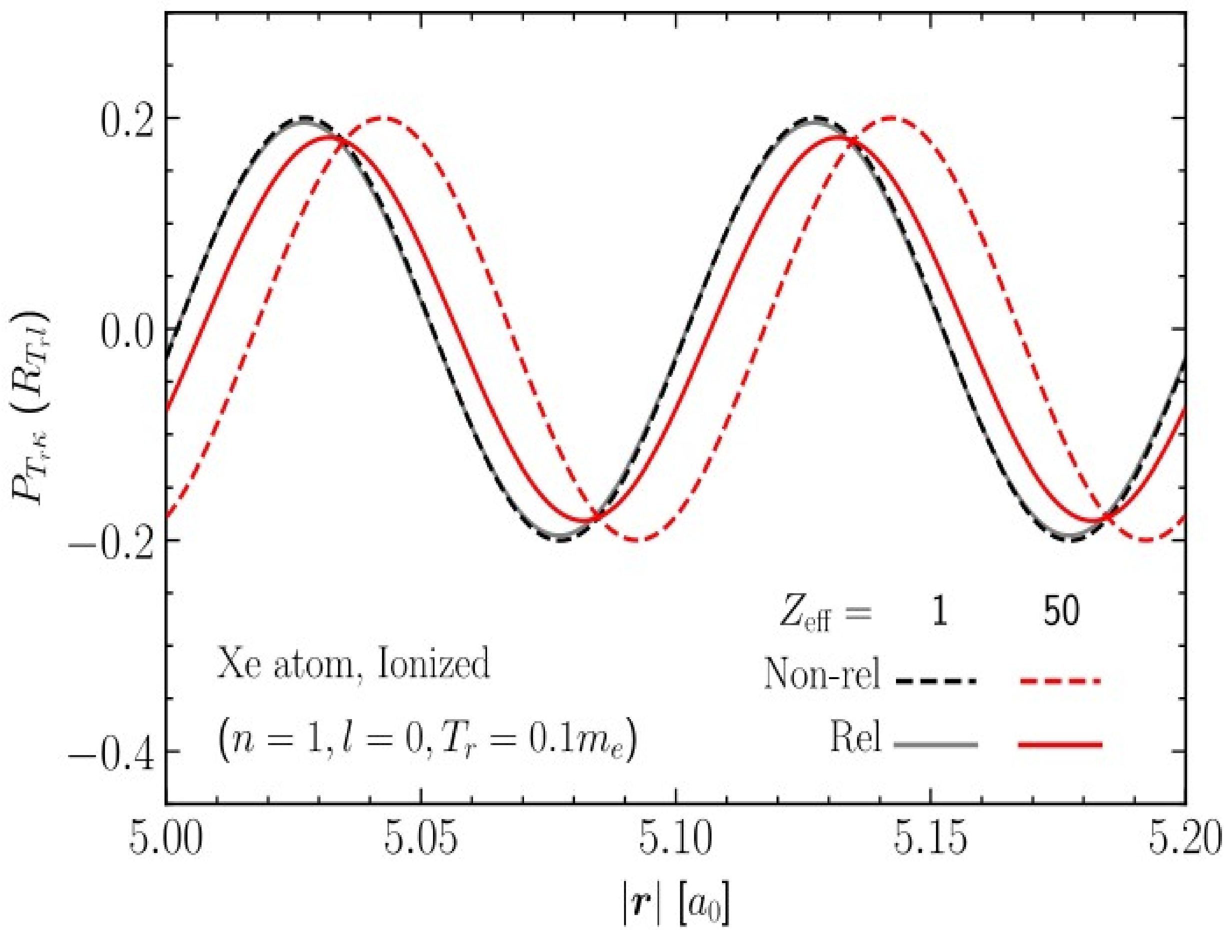
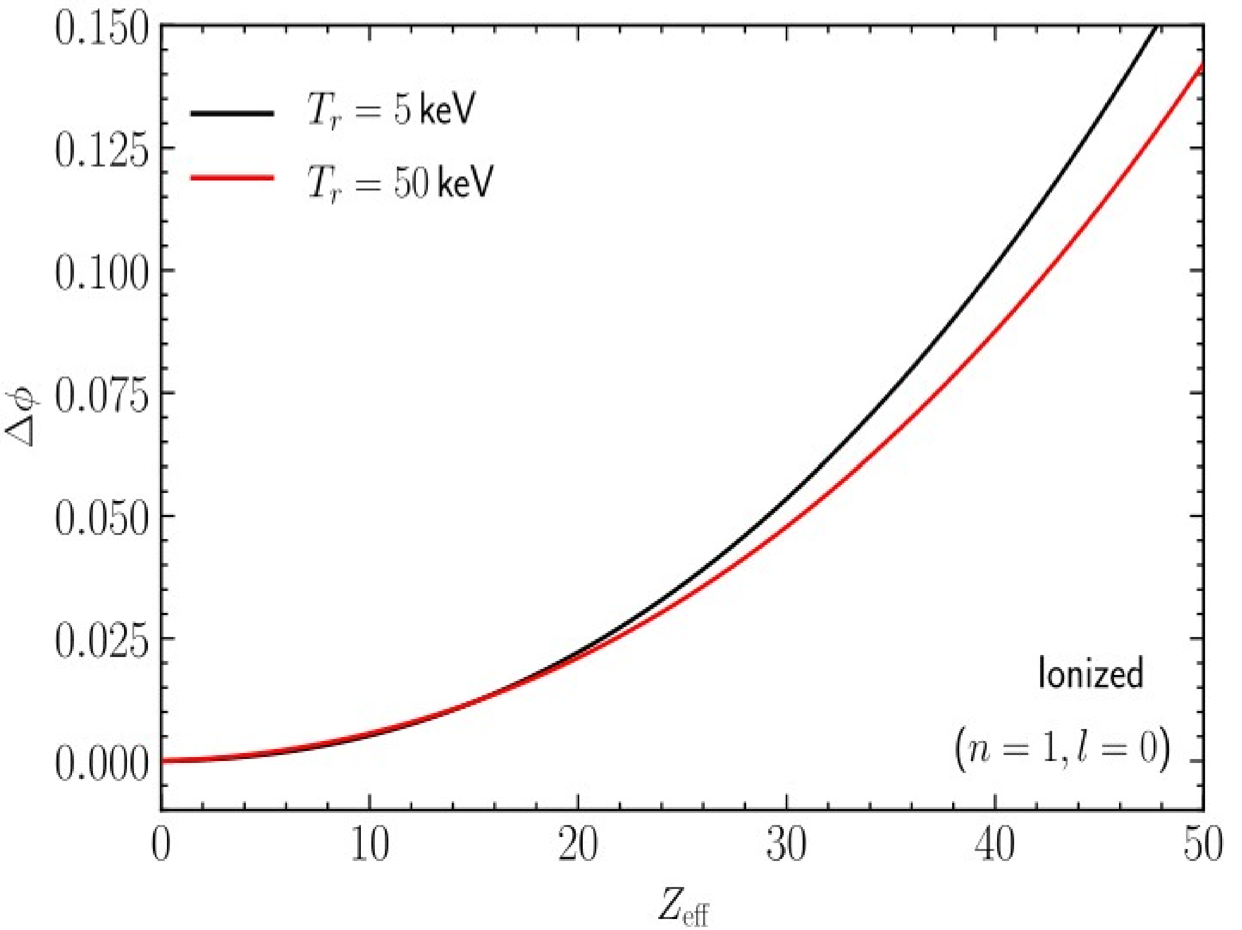
Similar oscillatory shape, but different phase!





Strong dependence on the effective charge $Z_{\rm eff}$

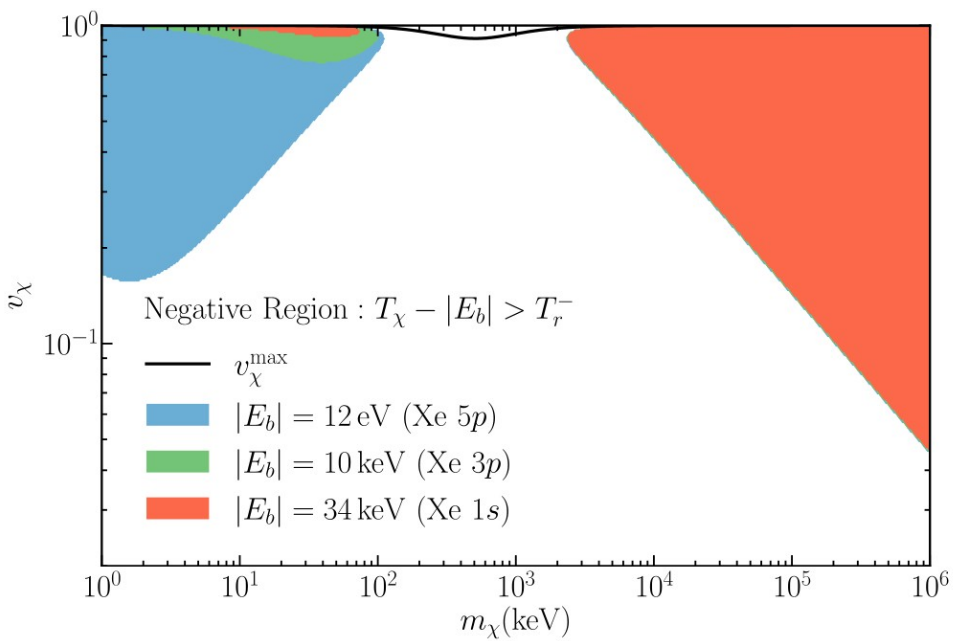
Summary

- **Factorization is not guaranteed!**

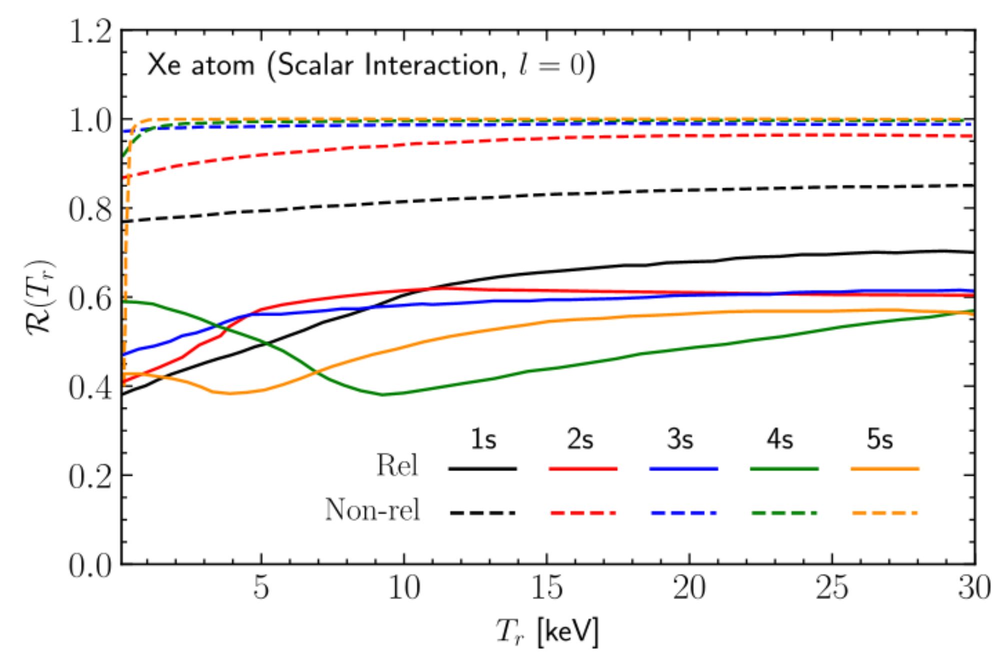
True only in the non-relativistic limit.

$$|\mathcal{M}|^2 \equiv |\mathcal{M}_{\text{free}}|^2 \times |f_{i\to f}(\boldsymbol{q})|^2 \quad \Rightarrow \quad |\mathcal{M}|^2 \equiv |\mathcal{M}_0(m_e)|^2 \times |f_{i\to f}(\boldsymbol{q})|^2$$

- **Negative Rate!**



- **Relativistic effect**



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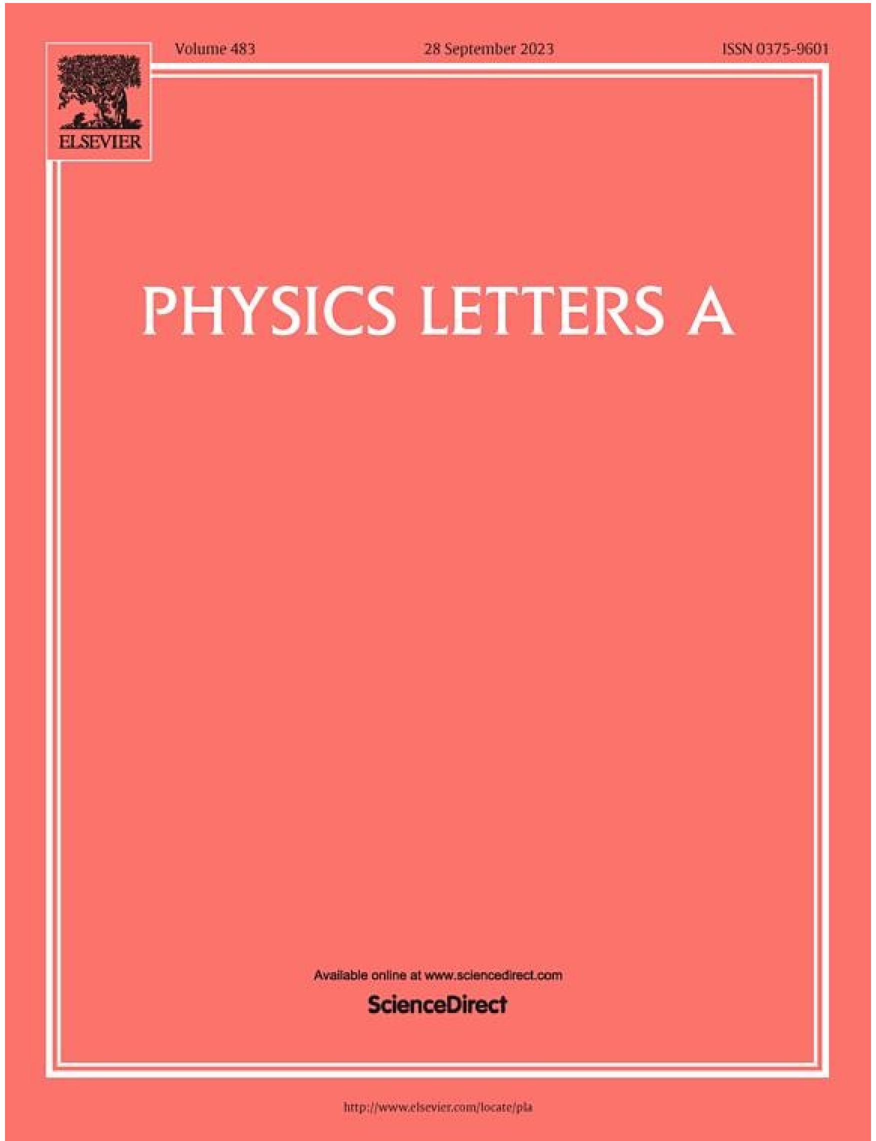
Thank You

Physics Letters A



Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.