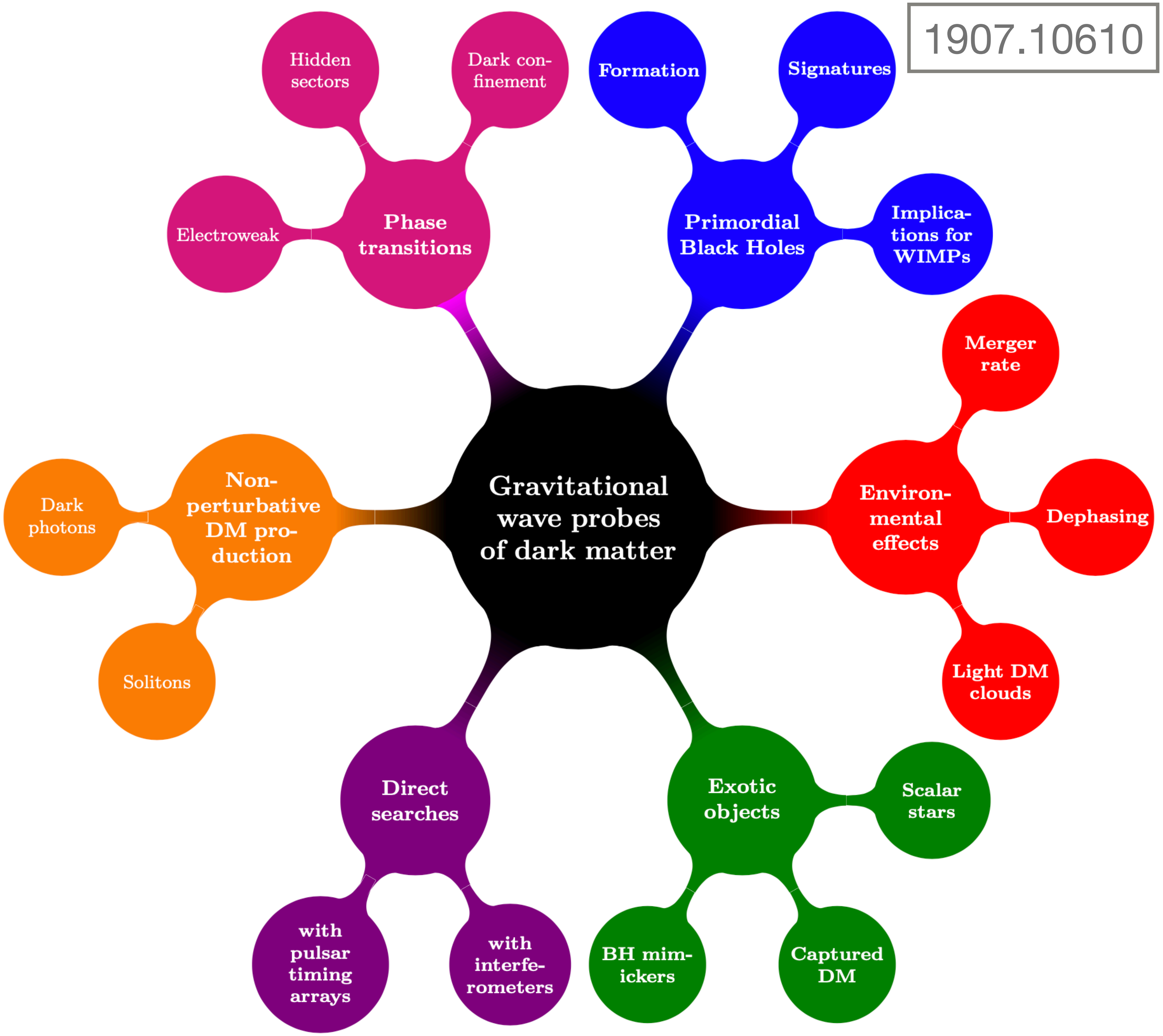
Gravitational waves from dark oscillations or bremsstrahlung

Wei Chao
Beijing Normal University

Modern strategies of probing dark matter



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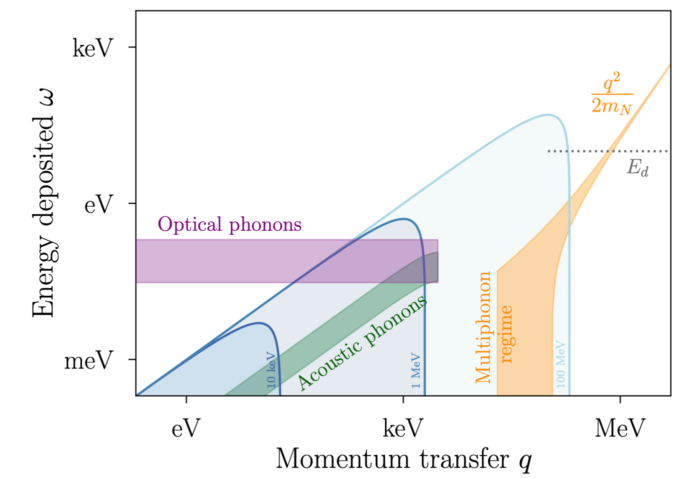
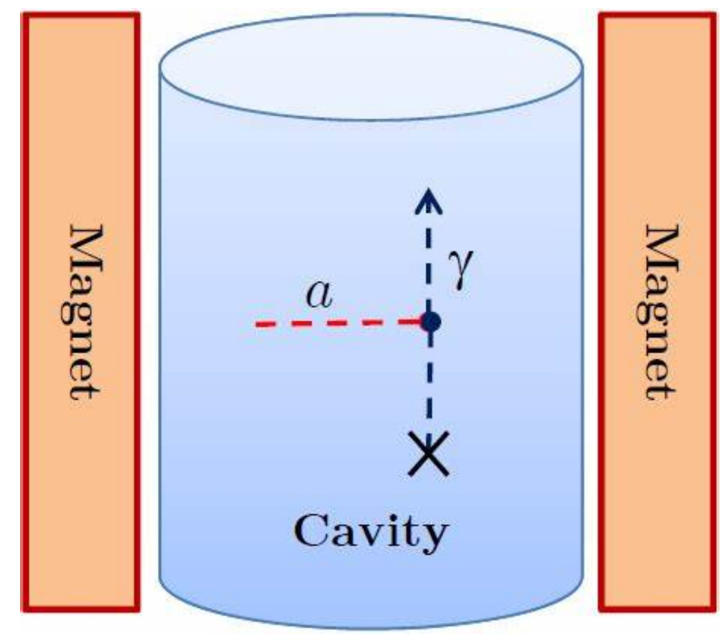




Outline

* GW spectrum from oscillation of dark photon
* GW from oscillation of ALP
* GW spectrum from graviton bremsstrahlung in DM production via Freeze-in

Ultralight DM oscillating into GWs

Dark photon DM

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_{pl}^2 R - \frac{1}{4}\hat{F}_{\mu\nu}g^{\alpha\mu}\hat{F}_{\alpha\beta}g^{\beta\nu} - \frac{\varepsilon}{2}F_{\mu\nu}g^{\alpha\mu}\hat{F}_{\alpha\beta}g^{\beta\nu}\right)$$

g: **Determinant of the background metric**

$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ **with** $|h_{\mu\nu} \ll 1|$

ε : **Mixing between photon and dark photon**

Equation of motion

$$\partial_\nu \hat{F}^{\mu\nu} = \hat{j}^\mu + \partial_\nu\left(-\frac{1}{2}h\hat{F}^{\mu\nu} + \hat{F}^{\mu\alpha}h_\alpha^\nu - \hat{F}^{\nu\alpha}h_\alpha^\mu\right) + \varepsilon\partial_\nu\left(-\frac{1}{2}hF^{\mu\nu} + F^{\mu\alpha}h_\alpha^\nu - F^{\nu\alpha}h_\alpha^\mu\right)$$

Ultralight DM oscillating into GWs

Dark photon DM

Schrodinger-like equation

$$\boxed{i\frac{\partial}{\partial z}\begin{pmatrix}\gamma_{\parallel}\\ \gamma'_{\parallel}\\ G_{\times}\end{pmatrix} = \left[\omega + \begin{pmatrix}\Delta_{\gamma\gamma} & \Delta_{\gamma\gamma'} & \Delta_{\gamma h}\\ \Delta_{\gamma\gamma'} & \Delta_{\gamma'\gamma'} & \Delta_{\gamma' h}\\ \Delta_{\gamma h} & \Delta_{\gamma' h} & 0\end{pmatrix}\right]\begin{pmatrix}\gamma_{\parallel}\\ \gamma'_{\parallel}\\ G_{\times}\end{pmatrix}}$$

$$\Delta_{\gamma\gamma} = \Delta_{\mathrm{pla}} + \Delta_{\mathrm{vac}} + \Delta_{\mathrm{CM}},$$

Plasma effect QED vacuum effect Cotton-Mouton birefringence

$$\Delta_{\gamma' h} = \frac{\varepsilon}{2}\kappa\left|B_T\right| + \frac{1}{2}\kappa\left|B'_T\right|,$$

Transverse to the GW propagation

$$\Delta_{\gamma'\gamma'} \approx \frac{m_{A'}^2}{2\omega},$$

DP mass

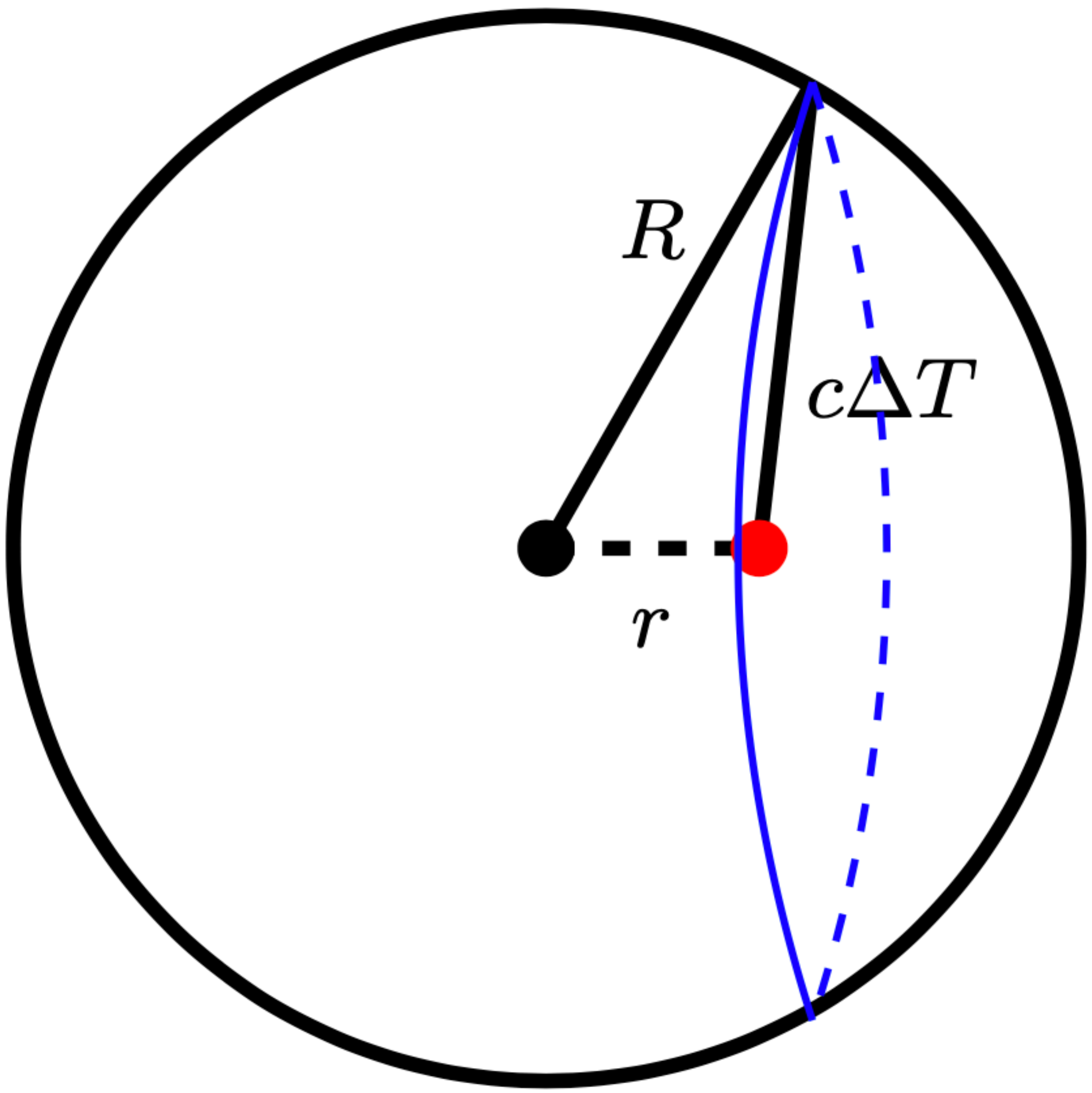
$$\Delta_{\gamma h} = \frac{1}{2}\kappa\left|B_T\right|,$$

Oscillation probability

$$P(i\to j) = \left|\sum_{\alpha}\mathcal{U}^*_{i\alpha}e^{-i\rho_\alpha z}\mathcal{U}_{j\alpha}\right|^2 , \quad i,j\in\gamma,\gamma',G_{\times}\,.$$

Ultralight DM oscillating into GWs

Dark photon DM: Local relic density



Survival probability in the galaxy cluster

$$f(r,\,\Delta T)=\begin{cases}1\,, & c\Delta T\le R-r\,,\\ \frac{1}{2}\left(1-\frac{r^2+c^2\Delta T^2-R^2}{2rc\Delta T}\right)\,, & c\Delta T>R-r\,,\\ 0\,, & c\Delta T\ge R+r\,,\end{cases}$$

Energy density

$$\rho_{\rm LGW}(t_0)=\frac{1}{V_{\rm GC}}\int_{t_0-2T}^{t_0}dt\int_0^R dr\mathscr{L}(r)\cdot f(r,t_0-t)4\pi r^2\,.$$

$$\mathscr{L}(r)=\rho_{\rm DM}(r)\mathscr{R}\quad {\rm GeV\cdot cm^{-3}\cdot s^{-1}}\,,$$

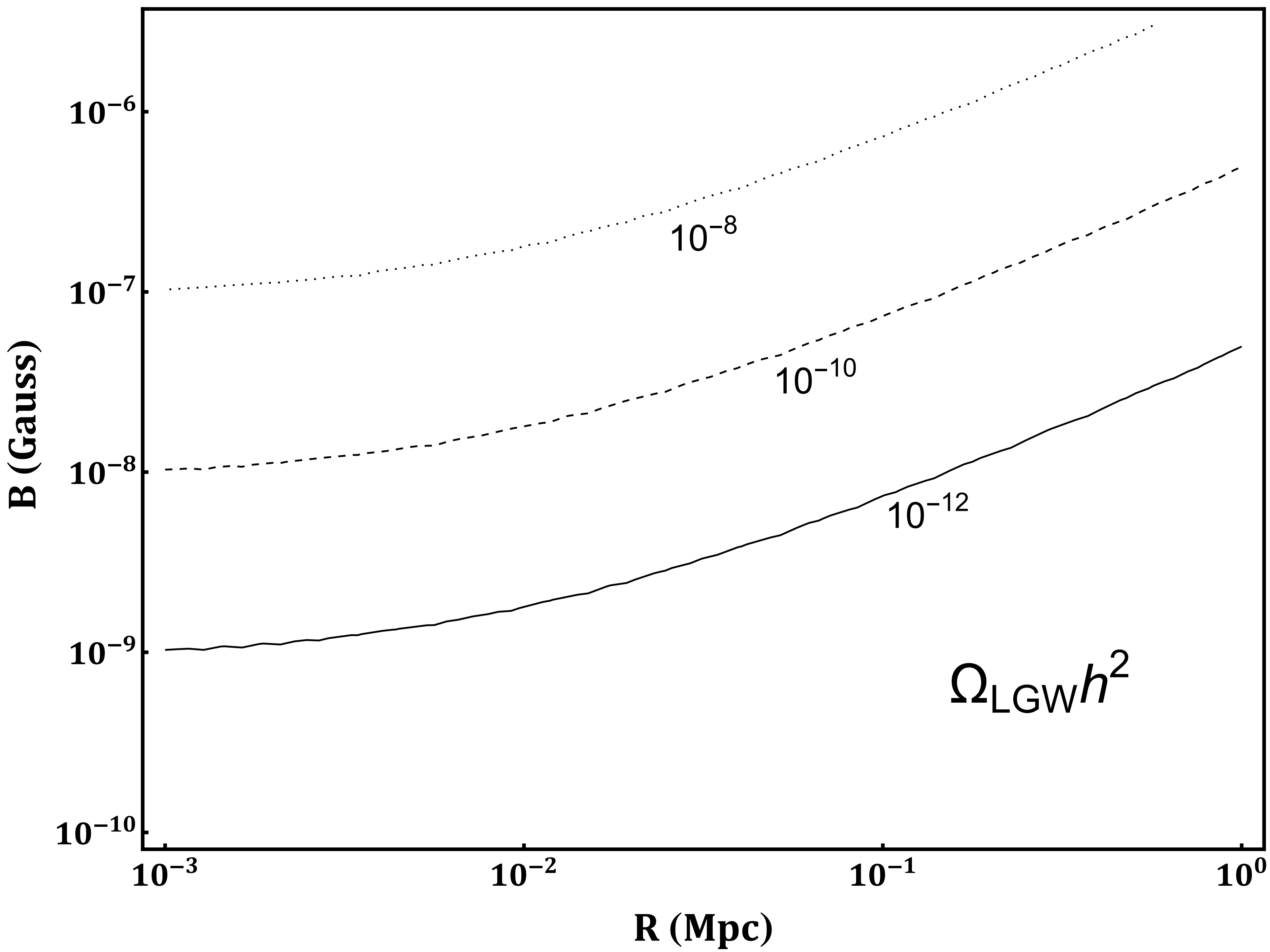
Luminosity of the oscillation

R: conversion rate per ULDP per second

GW flux $$\mathscr{D}_{\rm GW}=\int\frac{dn_{\rm DM}(r)}{dE}\frac{\mathscr{R}d^3r}{4\pi|\vec{r}-\vec{d}|^2}\sim 1.72\times10^{-7}/m_{\rm ULDP}^2\ {\rm cm^{-2}\cdot s^{-1}\cdot eV^{-1}}$$

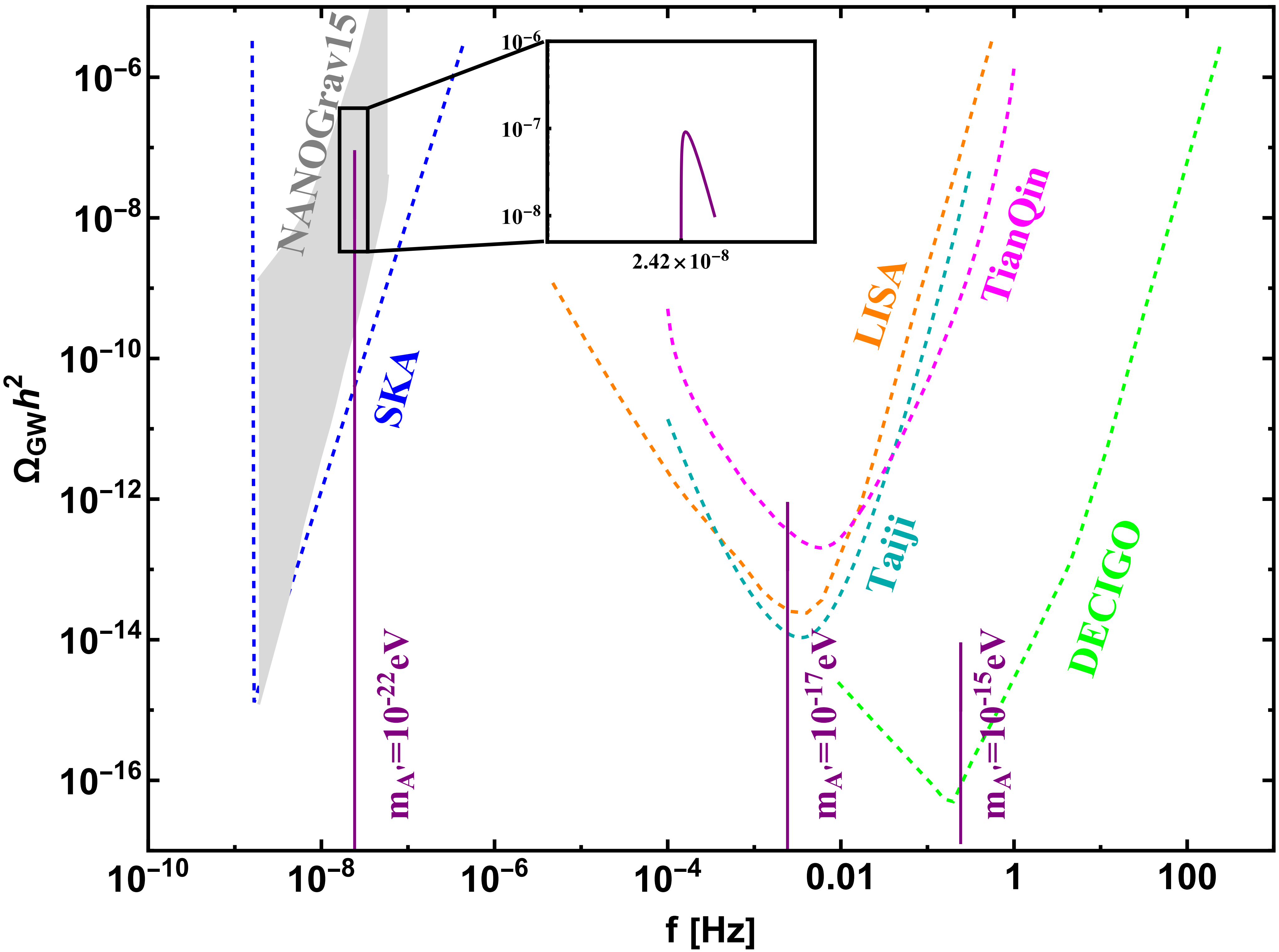
Ultralight DM oscillating into GWs

Dark photon DM



Ultralight DM oscillating into GWs

Dark photon DM



Ultralight DM oscillating into GWs

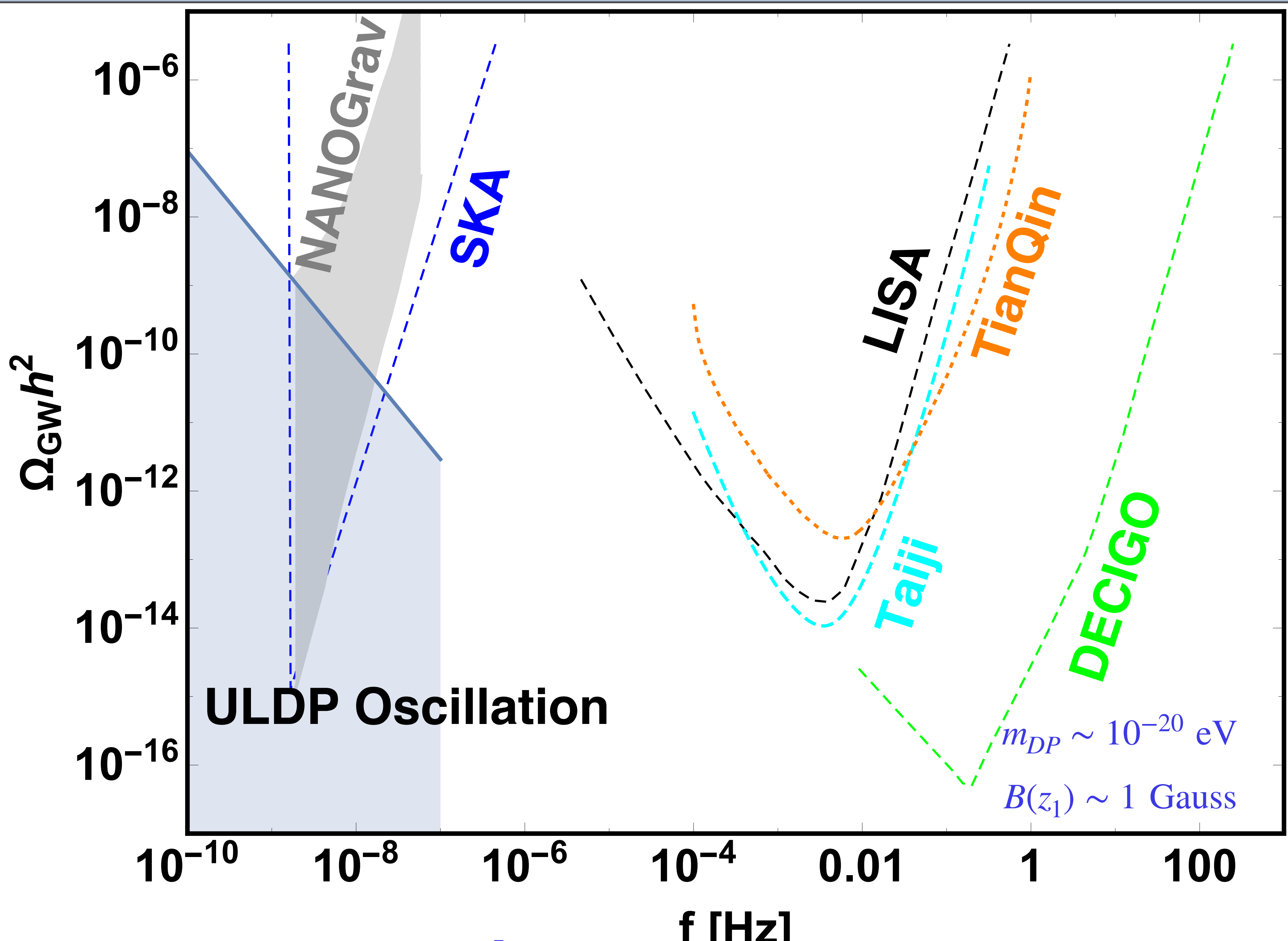
Dark photon oscillations in the early universe

Possible mechanisms for the production of primordial dark magnetic field

- PDMF can be generated during the pseudo-scalar inflation, ϕ, with the Chern-Simons interaction, $\frac{\phi}{4\pi f_a} F\widetilde{F}$.
- PDMF can be produced from the axion oscillations.
- PDMF can be generated from the oscillation of photon in the axion or some other exotic background.

Ultralight DM oscillating into GWs

Dark photon oscillations in the early universe



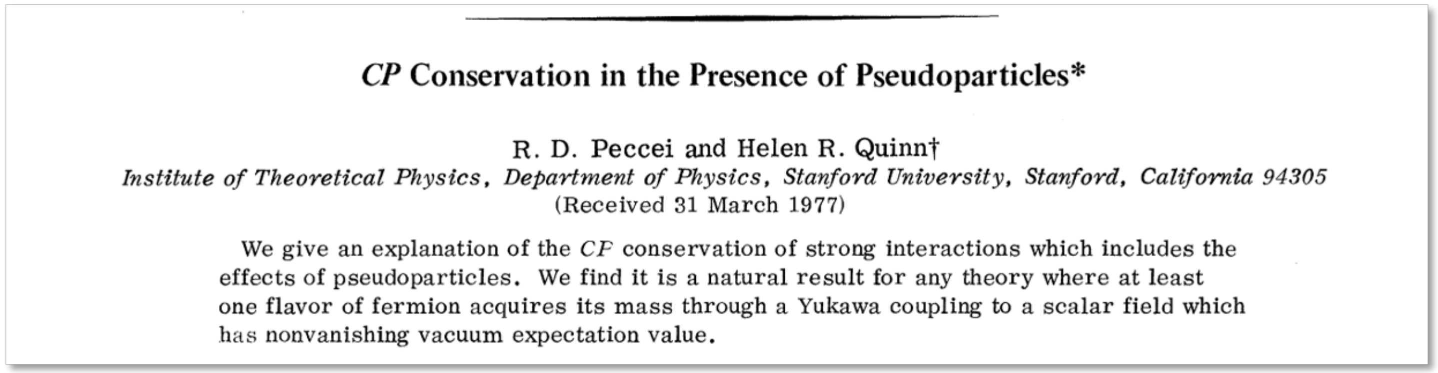
$$\rho_{\rm SGW} = \int_{z_0}^{z_1} n_{\rm ULDP}(z_0)\mathscr{R}(z) m_{\rm ULDP} \frac{dz}{(1+z)^2 H(z)}$$

Outline

- GW from oscillation dark photon
- GW from oscillation of axion-like particles
- GW from graviton bremsstrahlung in DM production via Freeze-in

Ultralight DM oscillating into GWs

Axion-like particle as DM



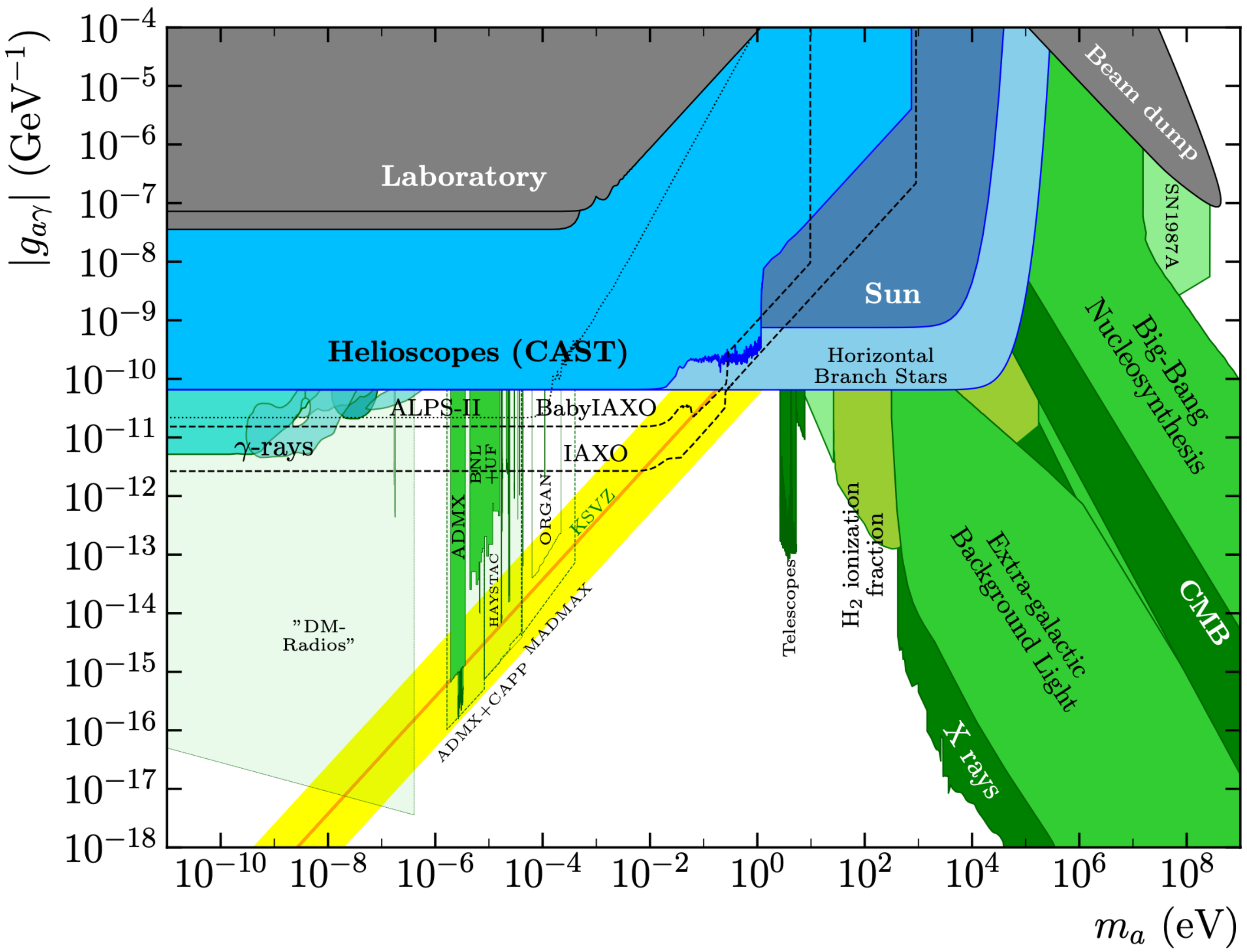
CP Conservation in the Presence of Pseudoparticles*

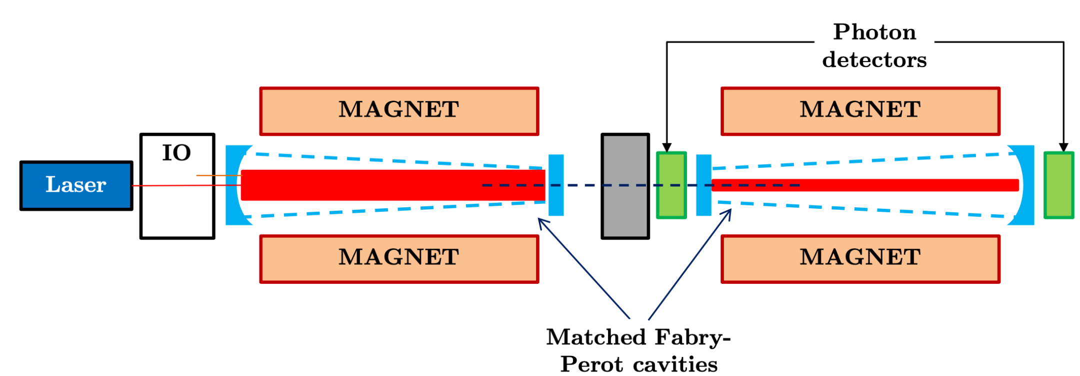
R. D. Peccei and Helen R. Quinn†

Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305

(Received 31 March 1977)

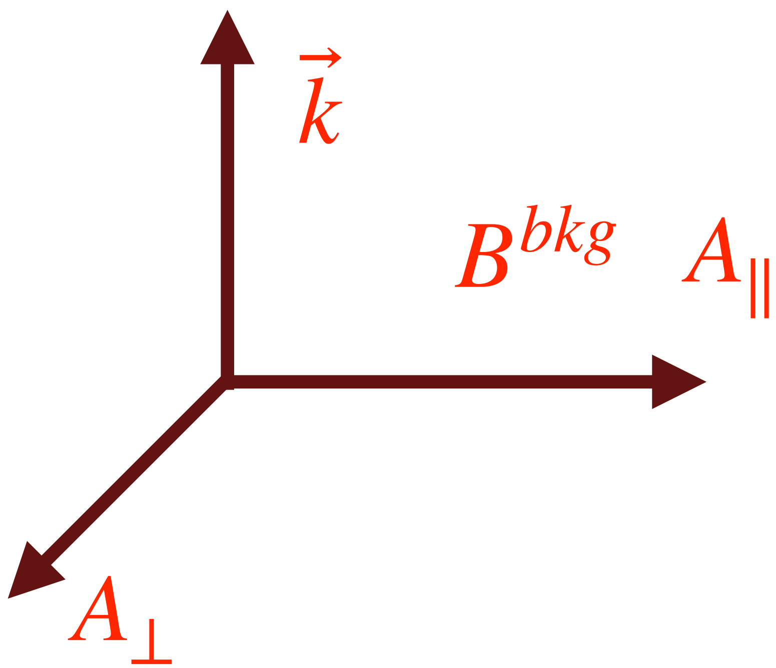
We give an explanation of the *CP* conservation of strong interactions which includes the effects of pseudoparticles. We find it is a natural result for any theory where at least one flavor of fermion acquires its mass through a Yukawa coupling to a scalar field which has nonvanishing vacuum expectation value.





Ultralight DM oscillating into GWs

Axion-like particle as DM



$$
\begin{aligned}
\Delta_{\gamma\gamma} &= \Delta_{\rm pla} + \Delta_{\rm vac} + \Delta_{\rm CM} \\
\Delta_a &= \frac{m_a^2}{2\omega} \\
\Delta_{\gamma h} &= \frac{1}{\sqrt{3}}\kappa B_T \\
\Delta_{a\gamma} &= \frac{g B_T}{\sqrt{3}}
\end{aligned}
$$

$$
\left(\omega - i\partial_z + \begin{pmatrix} 0 & \Delta_{\gamma h} & 0 & 0 & 0 \\ \Delta_{\gamma h} & \Delta_{\gamma\gamma}^{\perp} & 0 & \Delta_R & \Delta_{a\gamma} \\ 0 & 0 & 0 & \Delta_{\gamma h} & 0 \\ 0 & \Delta_R & \Delta_{\gamma h} & \Delta_{\gamma\gamma}^{\parallel} & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix}\right) \begin{pmatrix} G_\times^\omega \\ A_\parallel^\omega \\ G_+^\omega \\ A_\perp^\omega \\ a \end{pmatrix} = 0
\quad \Rightarrow \quad
i\partial_z \begin{pmatrix} G_\times^\omega \\ A_\parallel^\omega \\ a \end{pmatrix} = \begin{pmatrix} 0 & \Delta_{\gamma h} & 0 \\ \Delta_{\gamma h} & \Delta_{\gamma\gamma} & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix} \begin{pmatrix} G_\times^\omega \\ A_\parallel^\omega \\ a \end{pmatrix}
$$

Ultralight DM oscillating into GWs

A: $\Delta_a = 0$ in the early universe: Three texture zeros in the Hamilton

$$
\begin{pmatrix}
-\frac{\Delta_{\gamma a}}{\sqrt{\Delta_{\gamma a}^2+\Delta_{g\gamma}^2}} & 0 & \frac{\Delta_{\gamma h}}{\sqrt{\Delta_{\gamma a}^2+\Delta_{g\gamma}^2}} \\
\frac{\sqrt{2}\Delta_{\gamma h}}{\sqrt{\eta^2-\Delta_{\gamma\gamma}\eta}} & -\sqrt{\frac{\eta-\Delta_{\gamma\gamma}}{2\eta}} & \frac{\sqrt{2}\Delta_{\gamma a}}{\sqrt{\eta^2-\Delta_{\gamma\gamma}\eta}} \\
\frac{\sqrt{2}\Delta_{\gamma h}}{\sqrt{\eta^2+\Delta_{\gamma\gamma}\eta}} & -\sqrt{\frac{\eta+\Delta_{\gamma\gamma}}{2\eta}} & \frac{\sqrt{2}\Delta_{\gamma a}}{\sqrt{\eta^2+\Delta_{\gamma\gamma}\eta}}
\end{pmatrix}
\begin{pmatrix}
0 & \Delta_{\gamma h} & 0 \\
\Delta_{\gamma h} & \Delta_{\gamma\gamma} & \Delta_{\gamma a} \\
0 & \Delta_{\gamma a} & 0
\end{pmatrix}
\begin{pmatrix}
-\frac{\Delta_{\gamma a}}{\sqrt{\Delta_{\gamma a}^2+\Delta_{g\gamma}^2}} & 0 & \frac{\Delta_{\gamma h}}{\sqrt{\Delta_{\gamma a}^2+\Delta_{g\gamma}^2}} \\
\frac{\sqrt{2}\Delta_{\gamma h}}{\sqrt{\eta^2-\Delta_{\gamma\gamma}\eta}} & -\sqrt{\frac{\eta-\Delta_{\gamma\gamma}}{2\eta}} & \frac{\sqrt{2}\Delta_{\gamma a}}{\sqrt{\eta^2-\Delta_{\gamma\gamma}\eta}} \\
\frac{\sqrt{2}\Delta_{\gamma h}}{\sqrt{\eta^2+\Delta_{\gamma\gamma}\eta}} & -\sqrt{\frac{\eta+\Delta_{\gamma\gamma}}{2\eta}} & \frac{\sqrt{2}\Delta_{\gamma a}}{\sqrt{\eta^2+\Delta_{\gamma\gamma}\eta}}
\end{pmatrix}^T
$$

$$
= \begin{pmatrix}
0 & 0 & 0 \\
0 & \frac{1}{2}\left(\Delta_{\gamma\gamma} - \sqrt{4\Delta_{\gamma h}^2+\Delta_{\gamma\gamma}^2+4\Delta_{\gamma a}^2}\right) & 0 \\
0 & 0 & \frac{1}{2}\left(\Delta_{\gamma\gamma} + \sqrt{4\Delta_{\gamma h}^2+\Delta_{\gamma\gamma}^2+4\Delta_{\gamma a}^2}\right)
\end{pmatrix}
$$

$$
\Rightarrow \quad P(a \to h) \propto \frac{2\Delta_{\gamma h}^2}{\eta^2 - \eta\Delta_{\gamma\gamma}}
$$

Ultralight DM oscillating into GWs

A: $\Delta_a \neq 0$ in the early universe: Two texture zeros in the Hamilton

According to the method of Vista

$$\mathcal{U}^T . H . \mathcal{U} = \begin{pmatrix} \lambda - 2\sqrt{\frac{p}{3}}\cos\left(\frac{\pi}{3} - \frac{\varphi}{3}\right) & 0 & 0 \\ 0 & \lambda - 2\sqrt{\frac{p}{3}}\cos\left(\frac{\pi}{3} + \frac{\varphi}{3}\right) & 0 \\ 0 & 0 & \lambda + 2\sqrt{\frac{p}{3}}\cos\left(\frac{\varphi}{3}\right) \end{pmatrix}$$

$$\lambda = \frac{\Delta_{\gamma\gamma} + \Delta_a}{3} \qquad p = -\frac{(\Delta_{\gamma\gamma} + \Delta_a)^2}{3} + \Delta_\gamma \Delta_a - \Delta_{a\gamma}^2 - \Delta_{h\gamma}^2 \qquad q = -\lambda^3 + \lambda(\Delta_\gamma \Delta_a - \Delta_{a\gamma}^2 - \Delta_{h\gamma}^2) + \Delta_a \Delta_{h\gamma}^2$$

$$\mathcal{U} = \begin{pmatrix} \sqrt{\frac{\lambda_2\lambda_3(\Delta_a+\eta\lambda_1)}{\Delta_a(\lambda_1+\lambda_2)(\eta\lambda_1+\lambda_3)}} & \eta\sqrt{\frac{\lambda_1\lambda_3(\Delta_a-\eta\lambda_2)}{\Delta_a(\lambda_1+\lambda_2)(\lambda_3-\eta\lambda_2)}} & \sqrt{\frac{\lambda_1\lambda_2(\lambda_3-\Delta_a)}{\Delta_a(\eta\lambda_1+\lambda_3)(\lambda_3-\eta\lambda_2)}} \\ -\eta\sqrt{\frac{\lambda_1(\Delta_a+\eta\lambda_1)}{(\lambda_1+\lambda_2)(\eta\lambda_1+\lambda_3)}} & \sqrt{\frac{\lambda_2(\Delta_a-\eta\lambda_2)}{(\lambda_1+\lambda_2)(\lambda_3-\eta\lambda_2)}} & \sqrt{\frac{\lambda_3(\lambda_3-\Delta_a)}{(\eta\lambda_1+\lambda_3)(\lambda_3-\eta\lambda_2)}} \\ \eta\sqrt{\frac{\lambda_1(\lambda_3-\Delta_a)(\Delta_a-\eta\lambda_2)}{\Delta_a(\lambda_1+\lambda_2)(\eta\lambda_1+\lambda_3)}} & -\sqrt{\frac{\lambda_2(\lambda_3-\Delta_a)(\Delta_a+\eta\lambda_1)}{\Delta_a(\lambda_1+\lambda_2)(\lambda_3-\eta\lambda_2)}} & \sqrt{\frac{\lambda_3(\Delta_a+\eta\lambda_1)(\Delta_a-\eta\lambda_2)}{\Delta_a(\eta\lambda_1+\lambda_3)(\lambda_3-\eta\lambda_2)}} \end{pmatrix}$$

$$\varphi = \arccos\left(\frac{3^{3/2} q}{2 p^{3/2}}\right)$$

$$P_{(\alpha\to\beta)}(z) = \left| \sum_{j=1}^{3} \mathcal{U}_{\alpha j} e^{-iE_j z} \mathcal{U}^*_{\beta j} \right|^2$$

Second order interference induced DM-GW transition.

Outline

- GW from oscillation of dark photon
- GW from oscillation of ALP
- GW from graviton bremsstrahlung in DM production via Freeze-in

Freeze-in Mechanism

L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, JHEP 03, 080 (2010), arXiv:0911.1120 [hep-ph]

DM with negligible coupling with the plasma: $\mathscr{L} \supset -\beta\,\phi\,\bar{\psi}\psi,$

Boltzmann equation:

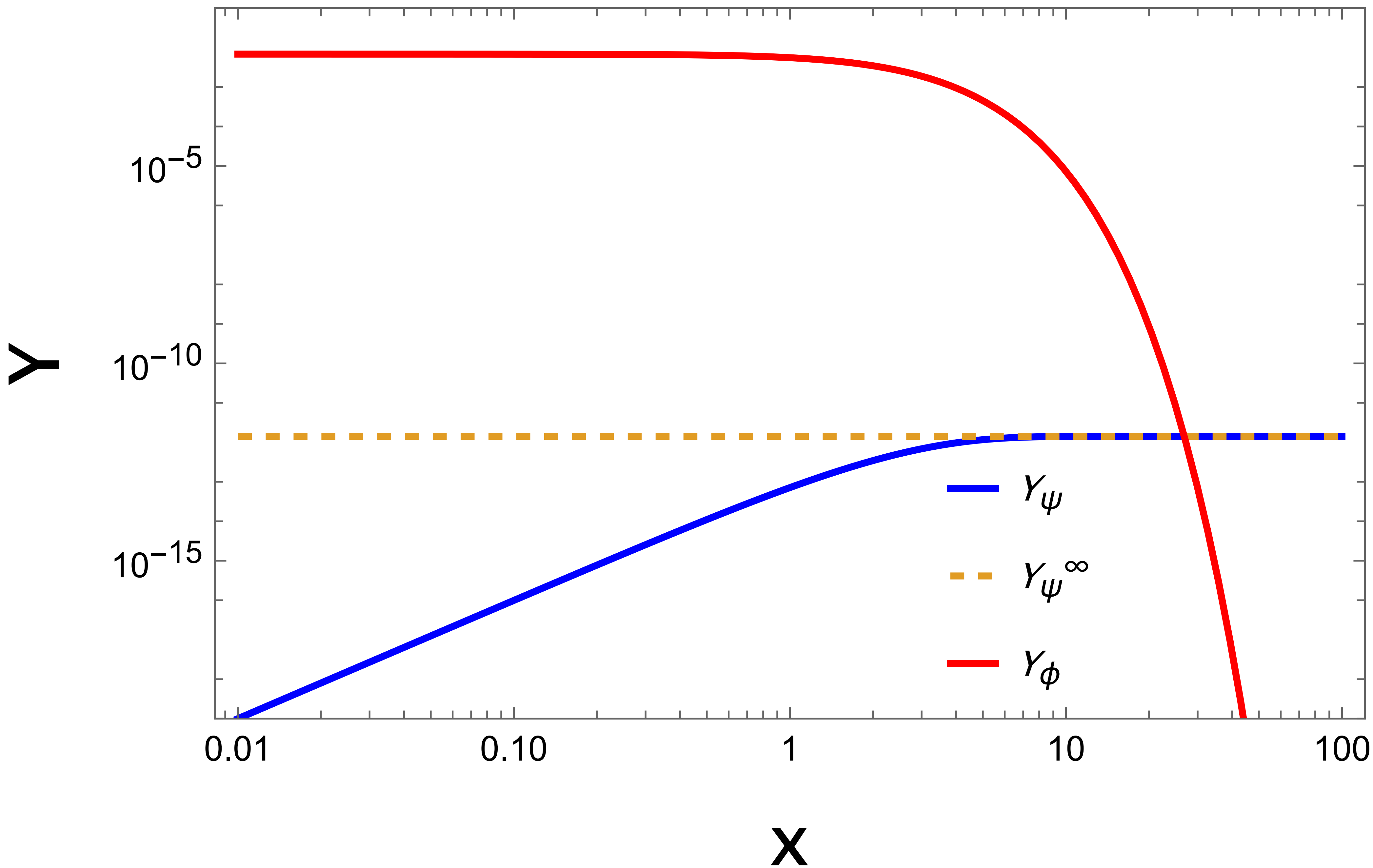
$$\frac{dn_\psi}{dt} + 3Hn_\psi = g_\phi \int d\Pi_\phi\, d\Pi_\psi\, d\Pi_{\bar{\psi}}\, (2\pi)^4 \delta^4(p_\phi - p_\psi - p_{\bar{\psi}})\, |\mathscr{M}|^2_{\phi\to\psi\bar{\psi}}\, f_\phi$$
$$= \frac{1}{2\pi^2} m_\phi^2 T\, K_1(m_\phi/T)\, \Gamma_{\phi\to\psi\bar{\psi}},$$

Boltzmann equation of comoving number density:

$$\frac{dY_\psi}{dx} = \frac{90\sqrt{90}}{(2\pi)^6} \frac{M_P}{\sqrt{g_*^\rho\, g_*^S}} \frac{\beta^2}{m_\phi} x^3 K_1(x), \longrightarrow Y_\psi^\infty = \frac{135\sqrt{90}}{2(2\pi)^5} \frac{M_P}{\sqrt{g_*^\rho\, g_*^S}} \frac{\beta^2}{m_\phi}.$$

Freeze-in Mechanism

L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, JHEP 03, 080 (2010), arXiv:0911.1120 [hep-ph]



Ultraviolet Freeze-in Mechanism

F. Elahi, C. Kolda, and J. Unwin, Journal of High Energy Physics 2015, 1 (2015).

DM with high dimensional coupling with the plasma: $\mathscr{L} \supset -\frac{1}{\Lambda}\chi\chi^\dagger \bar{f}f,$

Boltzmann equation:

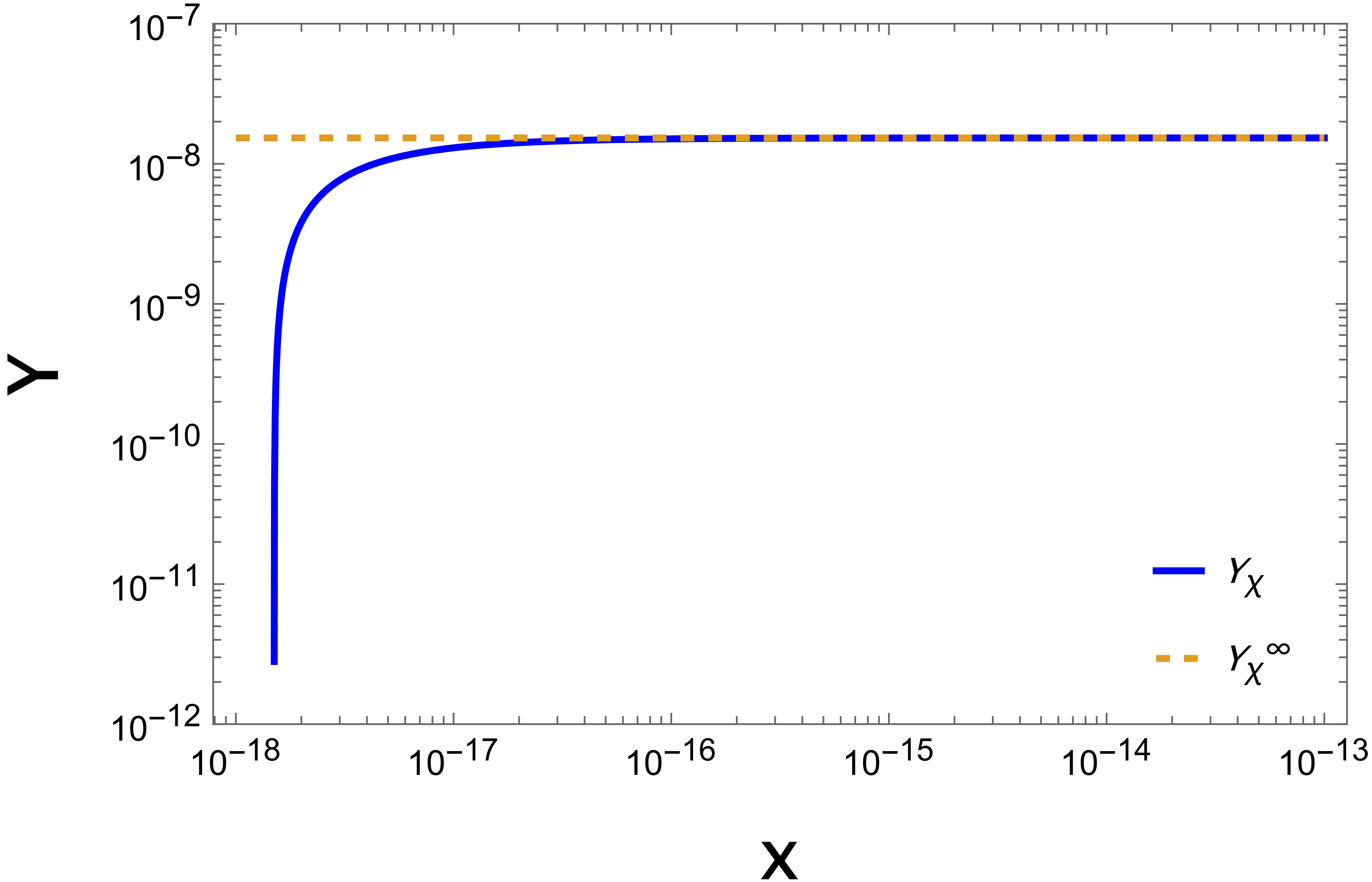
$$\frac{dn_\chi}{dt} + 3Hn_\chi = g_f g_{\bar{f}} \int d\Pi_f \, d\Pi_{\bar{f}} \, d\Pi_\chi \, d\Pi_{\chi^\dagger} \, (2\pi)^4 \delta^4(p_f + p_{\bar{f}} - p_\chi - p_{\chi^\dagger}) \, |\mathscr{M}|^2_{f\bar{f}\to\chi\chi^\dagger} \mathscr{F}_f \mathscr{F}_{\bar{f}},$$

For comoving number density:

$$\frac{dY_\chi}{dT} = -\frac{45\sqrt{90}}{2\pi^3(2\pi)^5} \frac{1}{g_*^S\sqrt{g_*^\rho}} \frac{M_P}{\Lambda^2} \longrightarrow Y_\chi^\infty \simeq \frac{180\sqrt{90}}{(2\pi)^8} \frac{1}{g_*^S\sqrt{g_*^\rho}} \frac{M_P T_R}{\Lambda^2}$$

Ultraviolet Freeze-in Mechanism

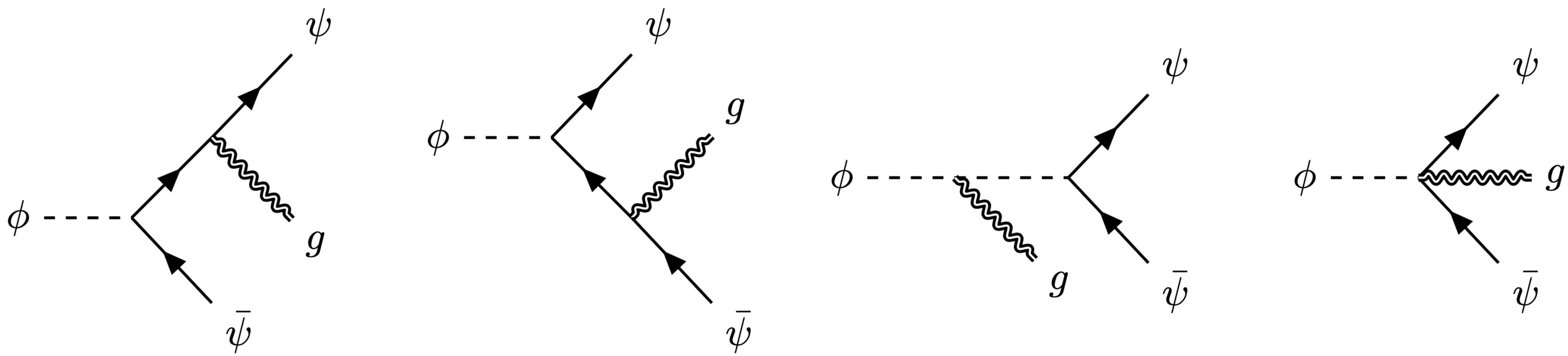
F. Elahi, C. Kolda, and J. Unwin, Journal of High Energy Physics 2015, 1 (2015).



1. A nice mechanism to address the DM production; 2. Difficult to be tested in the laboratory

Graviton bremsstrahlung in the freeze-in process

IR freeze-in case:



Boltzmann equation:

$$\frac{d\rho_{GW}}{dt} + 4H\rho_{GW} = g_\phi \int d\Pi_\phi d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_g (2\pi)^4 \delta^4(p_\phi - p_\psi - p_{\bar{\psi}} - p_g) \, |\mathcal{M}|^2 f_\phi \omega$$

$$\frac{d}{dx}\left(\frac{dY_{GW}}{d\omega}\right) = \frac{T^2}{m_\phi}\frac{1}{HTS^{4/3}}\frac{dC_{\phi\to\psi\bar{\psi}g}}{d\omega} = \left(\frac{45}{2\pi^2 g_*^S(T)}\right)^{4/3} \frac{\beta^2\sqrt{90}}{4(2\pi)^6\sqrt{g_*^\rho(T)}} \frac{\omega_D^2}{M_P T_D^2}$$

$$\times K_1(x)\, x^2 \left(1 - \frac{2\omega_D}{T_D}\frac{1}{x}\right)\left[2 - \frac{2T_D}{\omega_D}x + \left(\frac{T_D}{\omega_D}x\right)^2\right]$$

Graviton bremsstrahlung in the freeze-in process

IR freeze-in case:

Redshift of graviton

$$\omega(T) = \omega_D \frac{a(T_D)}{a(T)} = \left(\frac{g_*^S(T)}{g_*^S(T_D)} \right)^{1/3} \frac{T}{T_D} \omega_D$$

GW Spectrum

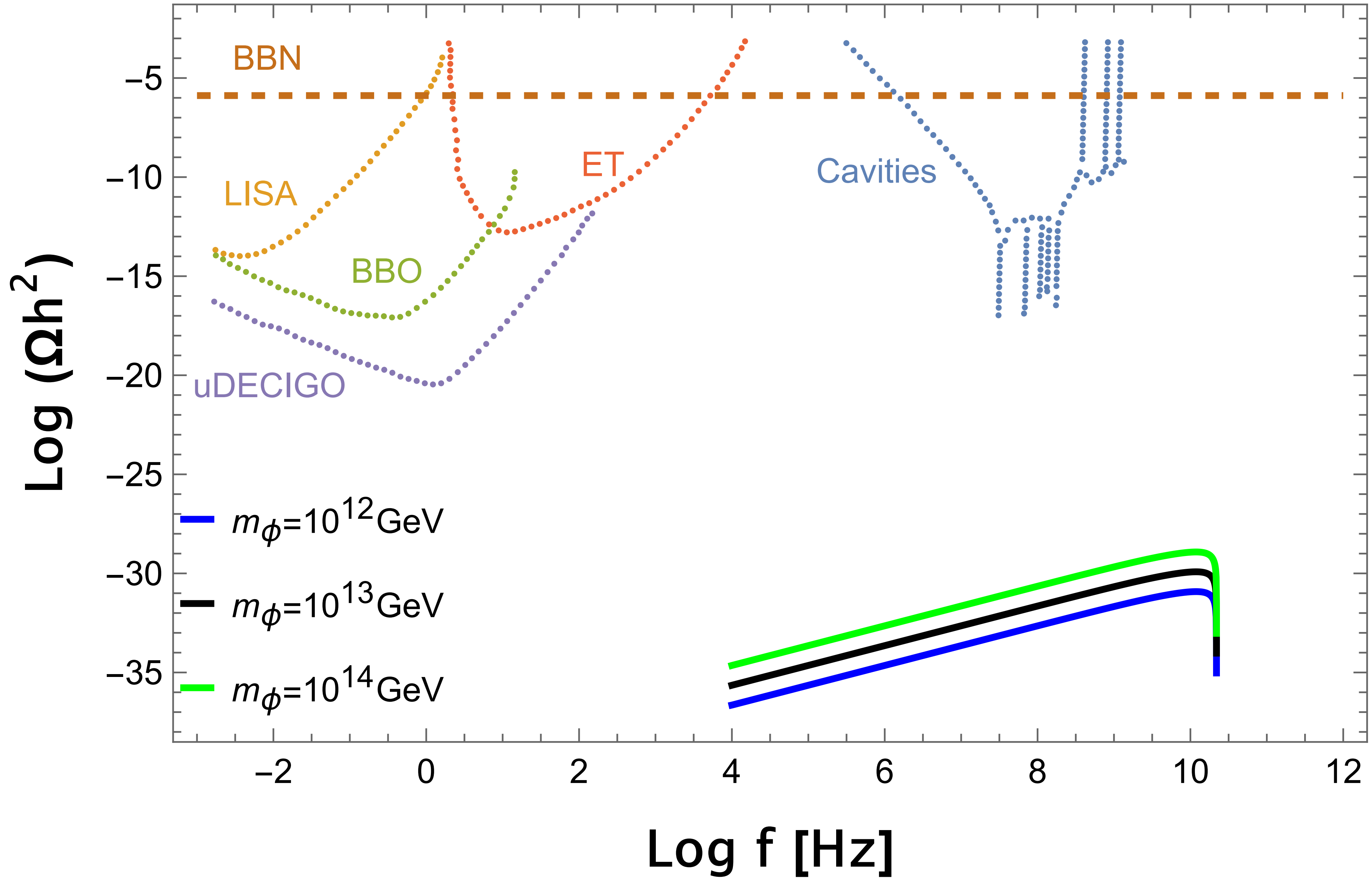
$$\Omega_{GW}(f) \equiv \frac{1}{\rho_{cr,0}} \frac{d\rho_{GW}(T_0)}{d \ln \omega_0} = \frac{1}{\rho_{cr,0}} \frac{dY_{GW}^{\infty}}{d \ln \omega_D} S^{4/3}(T_0)$$

$$= \frac{T_0^4}{\rho_{cr,0}} \frac{g_*^S(T_0)}{g_*^S(T_D)} \frac{\sqrt{90}\beta^2 T_D}{2(2\pi)^5 \sqrt{g_*^\rho(T_D)} M_P} \frac{f}{T_0} \times$$

$$\left[8 - 6\pi^2 \frac{f}{T_0} \left(\frac{g_*^S(T_D)}{g_*^S(T_0)} \right)^{1/3} + 24\pi^2 \frac{f^2}{T_0^2} \left(\frac{g_*^S(T_D)}{g_*^S(T_0)} \right)^{2/3} - 8\pi^4 \frac{f^3}{T_0^3} \frac{g_*^S(T_D)}{g_*^S(T_0)} \right]$$

Graviton bremsstrahlung in the freeze-in process

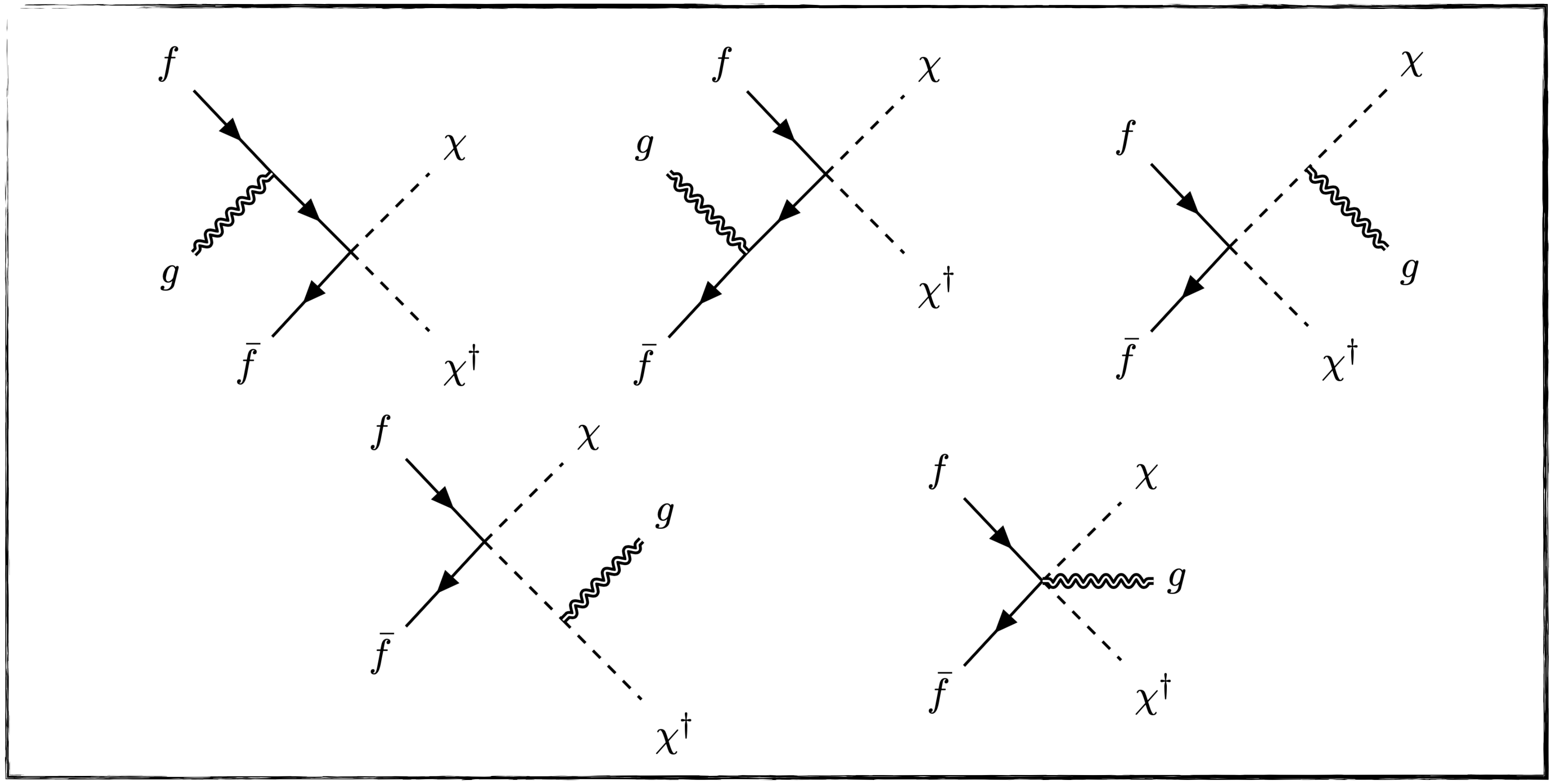
IR freeze-in case:



Max frequency: $$f_{max} = \frac{1}{2\pi}\omega_D^{max}\frac{a(T_D)}{a(T_0)} = \frac{1}{2\pi}\frac{m_\phi}{2}\left(\frac{g_*^S(T_0)}{g_*^S(T_D)}\right)^{1/3}\frac{T_0}{T_D} \simeq 2.3\times 10^{11}\mathrm{Hz}$$

Graviton bremsstrahlung in the freeze-in process

UV freeze-in case:



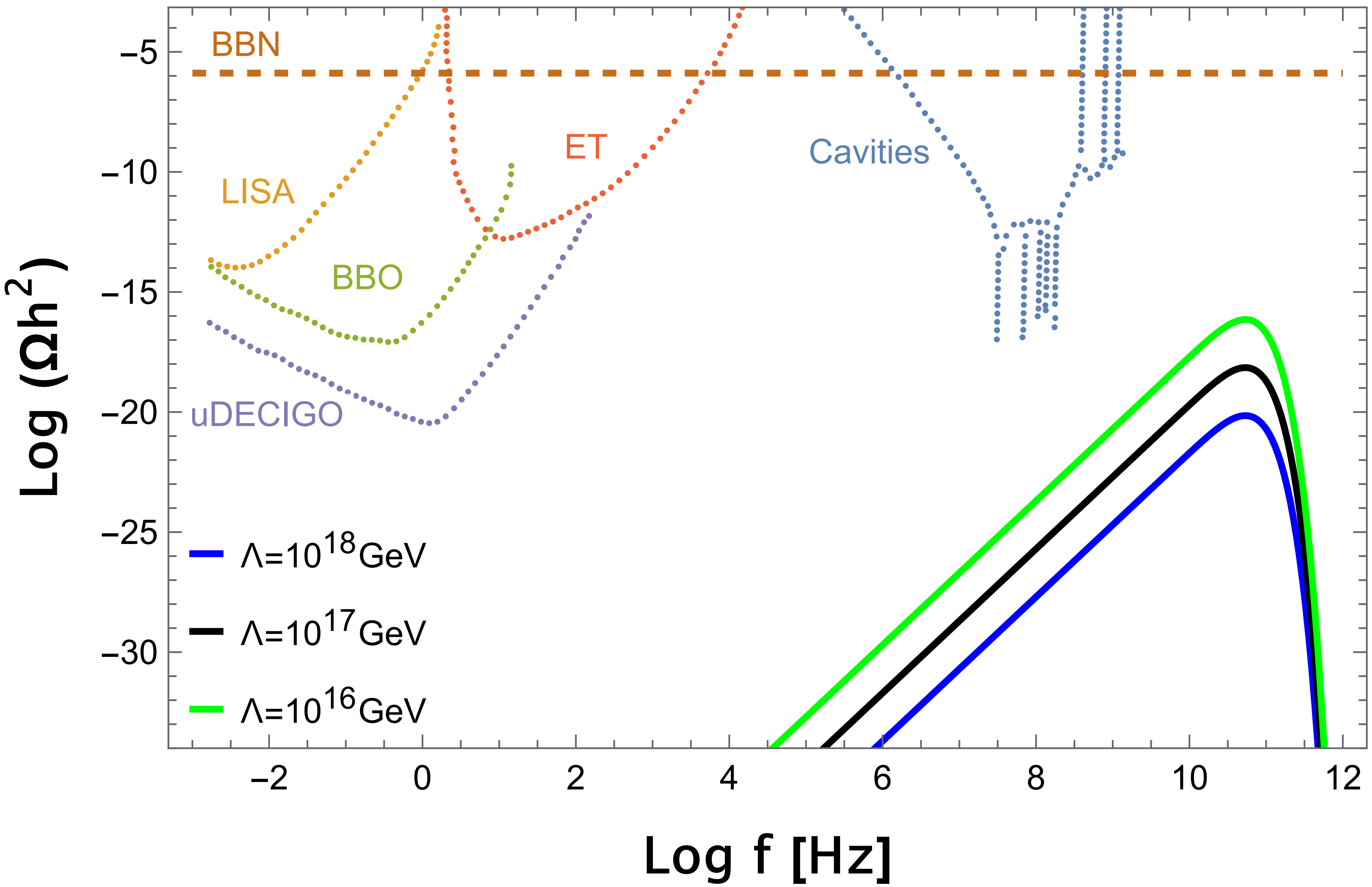
Differential collision term for GWs:

$$\frac{d}{d\omega}C_{f\bar{f}\to\chi\chi^\dagger g} = \frac{1}{(2\pi)^7}\frac{T^6\omega^2}{M_P^2\Lambda^2}G^{3,0}_{1,3}\left(\frac{\omega^2}{T^2}\middle|\begin{matrix}1\\0,2,3\end{matrix}\right)$$

$$\frac{dY^{\infty}_{GW}}{d\omega_E} = \left(\frac{45}{2\pi^2 g_*^S(T_E)}\right)^{4/3}\frac{\sqrt{90}}{\sqrt{g_*^\rho(T_E)}}\frac{1}{(2\pi)^8}\frac{\omega_E^2 T_R^2}{M_P\Lambda^2 T_E^2}G^{3,0}_{1,3}\left(\frac{\omega_E^2}{T_E^2}\middle|\begin{matrix}1\\0,2,3\end{matrix}\right)$$

Graviton bremsstrahlung in the freeze-in process

UV freeze-in case:



Spectrum at peak frequency $\Omega_{GW}h^2(f_{peak}) = 7.1 \times 10^{-17} \left(\frac{T_R}{10^{16}\,\mathrm{GeV}}\right)^3 \left(\frac{\Lambda^2}{10^{32}\,\mathrm{GeV}^2}\right)^{-1}$

Conclusion

We have calculated GW spectrum arising from dark matter related process:

- **Nano-hertz GW can be induced from oscillation of superlight DM.**

- **O(1) Hz GW can be induced from WIMP triggered phase transition.**

- **High frequency GW can be induced from graviton bremsstrahlung of DM production process.**

More systematic study is needed to build connection between dark matter and GW.

Thank you for your attention