

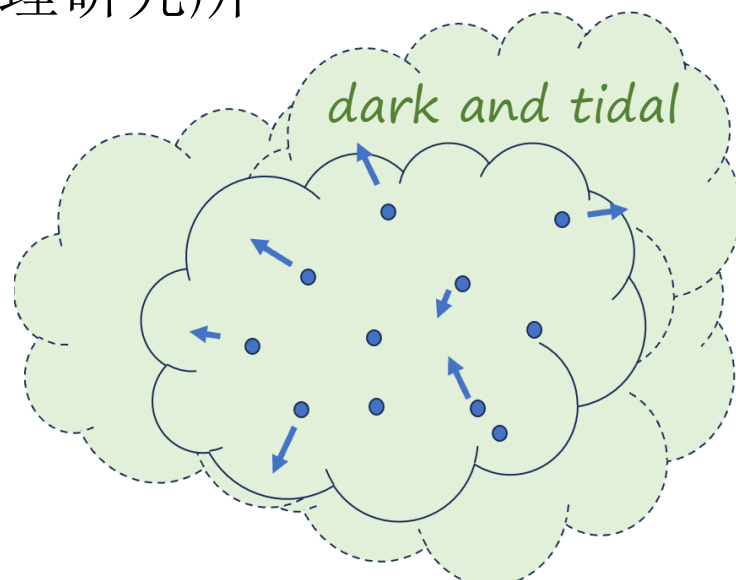
Axion Star Tidal Effects & Bosenova

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Z.Wang and Y. Gao, Phys.Rev.D 111 (2025) 4, 043042
Z.Wang and Y. Gao, 2508.14535

烟台 2025/10/19



- Brief remarks
- DM gravity on stars: kinetic heating
- Binary disruption: form factor for fuzzy objects
- Quick story about axion & DM
- Dilute stars
- Binary limits for axion stars & miniclusters.
- Bosenova in Miniclusters

Axion can form miniclusters & stars

Sensitivity on via density/gravity
structural patterns can shed some
light on boson DM properties.

Miniclusters:

Post-inflationary scenario causes inhomogeneities.
A naïve estimate on the clump masses:

$$M \approx \frac{4\pi}{3} (1 + \delta) \bar{\rho} H(T_{osc})^{-3}$$

Or solve the density fluctuation's equation

$$\ddot{\delta} + 2H\dot{\delta} + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G_N \bar{\rho}_a \right) \delta = 0$$

$$c_s^2 \approx \frac{k^2}{4m_a^2 a^2} \quad \text{See: } \begin{matrix} 1404.1938 & 1810.11468 \\ 1911.07853 & 2006.08637 \end{matrix}$$

1207.3124

Or use N-body simulation.

1911.09417 2101.04177
2207.11276 2402.18221

Axion stars:

See review:

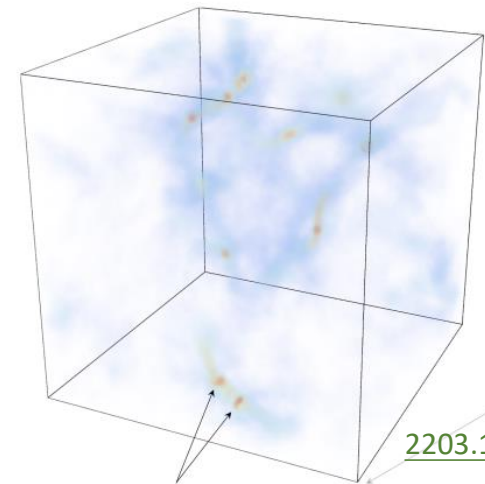
[Braaten & Zhang, 19'](#)
[J. Niemeyer, 19'](#)

Localized (soliton) solutions under
self-interaction / gravity

Oscillons ($\dot{m}_a > 0$): [astro-ph/9311037](#)

Boson star (w gravity) [1406.6586](#)

$$i\dot{\psi} = -\frac{\nabla^2 \psi}{2m} - Gm^2 \psi \int d^3x' \frac{\psi^*(\mathbf{x}')\psi(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{\partial}{\partial \psi^*} V_{nr}(\psi, \psi^*)$$



Solitons

[2203.10100](#)

Gravitational interests in dark clumps/solitons

All evidences of DM are gravitational: should gravity tell us more?

>> Preferred as a cored DM halo <<

Galaxy scale dynamics:

Disk thickening, stellar streams

Church, J. P. Ostriker, and P. Mocz, 18'

Amorisco and A. Loeb, 18'

excludes $m < 10^{-22}$ eV

Granularity above the de Broglie wavelength $\sim 2\pi/mv$ L.Hui, 16'

exclusion limit $m \rightarrow 10^{-21}$ eV

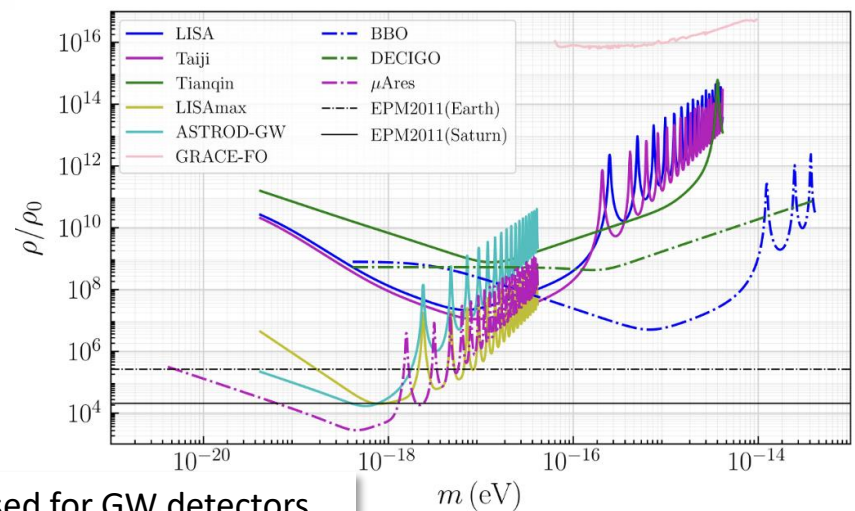
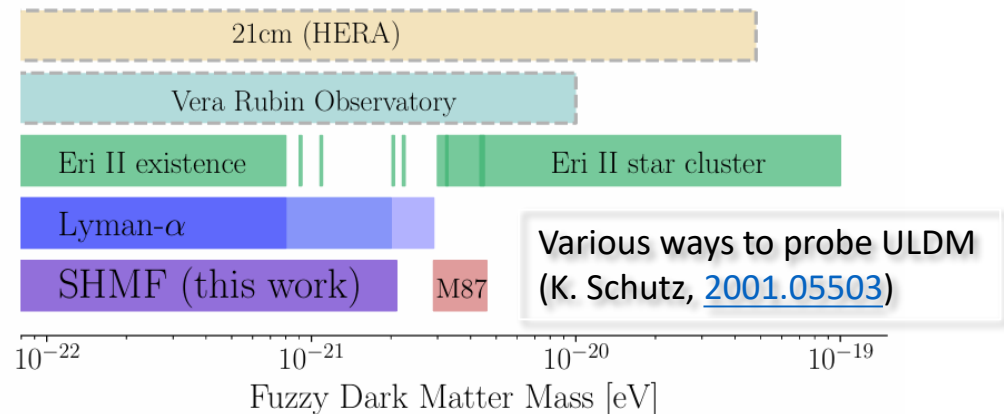
Relaxation of old clusters, exclusion

limit $m \rightarrow 10^{-20} \sim 10^{-19}$ eV

Bar-Or, Fouvry, and Tremaine, 19'

Marsh and Niemeye, 19'

Wasserman, 19' etc.



Proposed for GW detectors
(summary from [2404.04333](#))

The tidal relaxation

1. Density granularity (by solitons) above its coherent scale creates a randomized, noise-like gravitational potential.

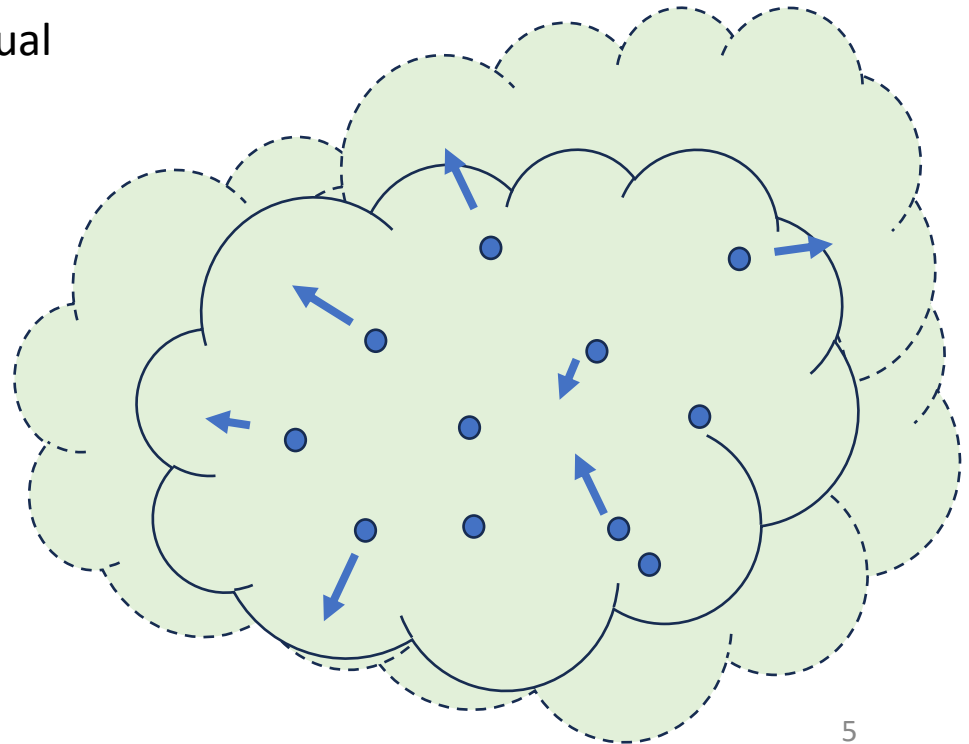
2. Randomized tidal perturbations drive stars away from their orbits, leading to eventual evaporation of the system

3. Most effective on systems comparable or larger than the granularity size.

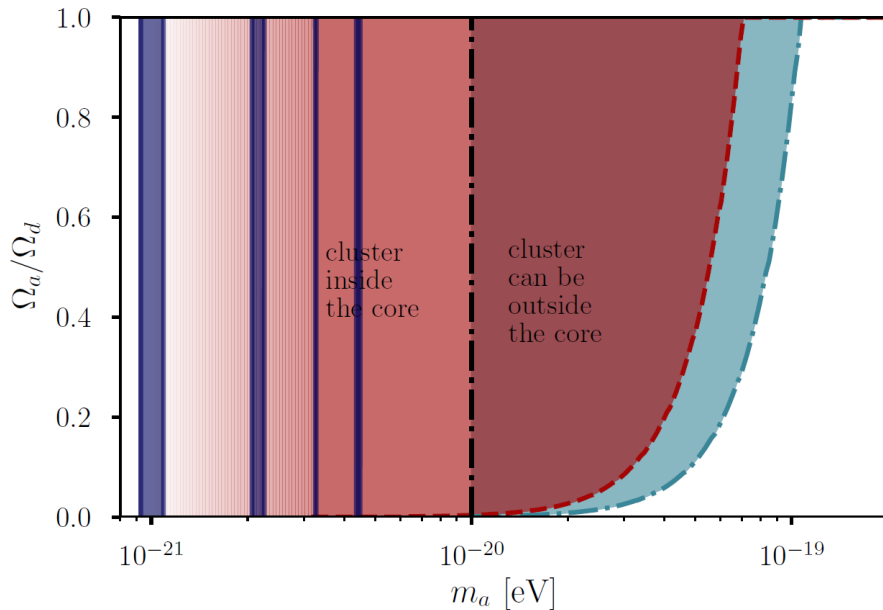
4. Unlike WIMPs, solitons have a macroscopically significant mass and contribute to relaxation.

DM granules ($\lambda \sim (mv)^{-1}$) passing by a star perturb the star's velocity, raising the (average) stellar velocity dispersion:

$$\Delta v_{\star}^2 \propto \left(\frac{v_{\star}}{v_{DM}} \right)^4 \frac{t}{m_{DM}^3 r_{1/2}^4}$$

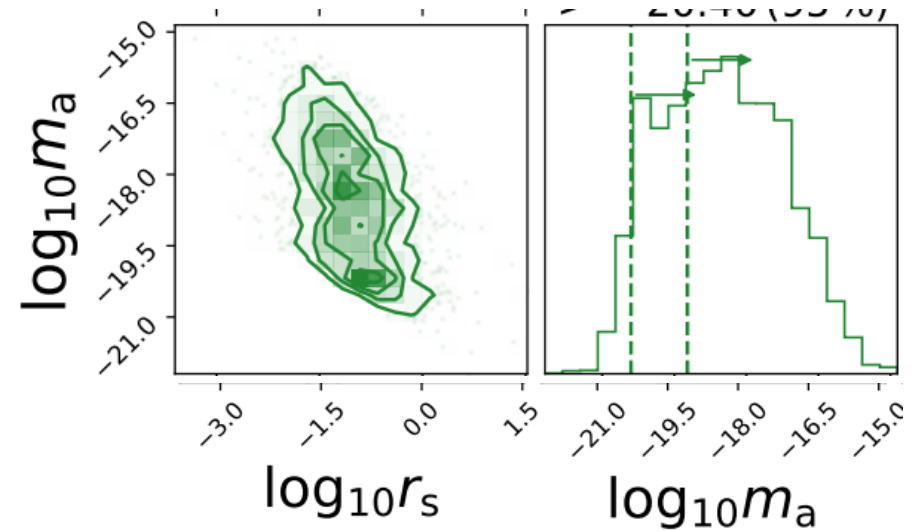


DM kinetic heating



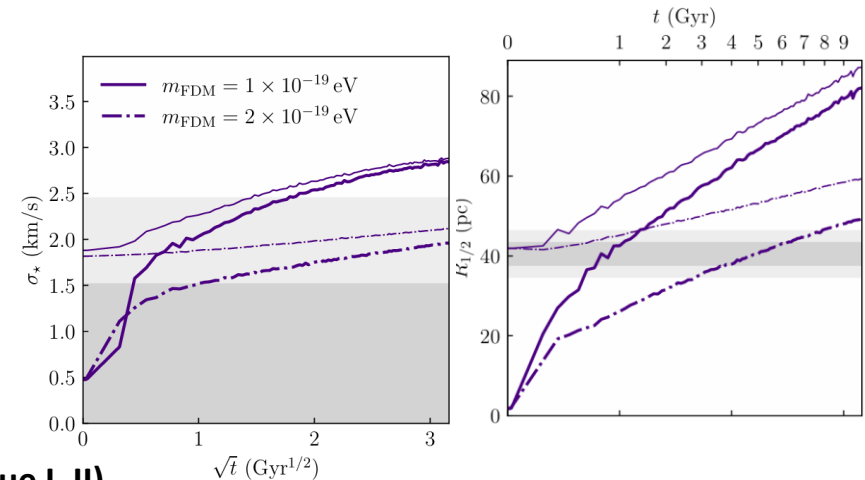
Axion DM granularity contributes to Cluster Relaxation

Relaxation for **Eridanus II**, see [Marsh and Niemeyer, 19'](#)
See 'revised constraints', [Chiang et.al: 2104.13359](#)



The MUSE-Faint survey ([2101.00253](#))

Quote: These limits are equivalent to a fuzzy-dark-matter particle mass $m_a > 4 \cdot 10^{-20} \text{ eVc}^{-2}$.

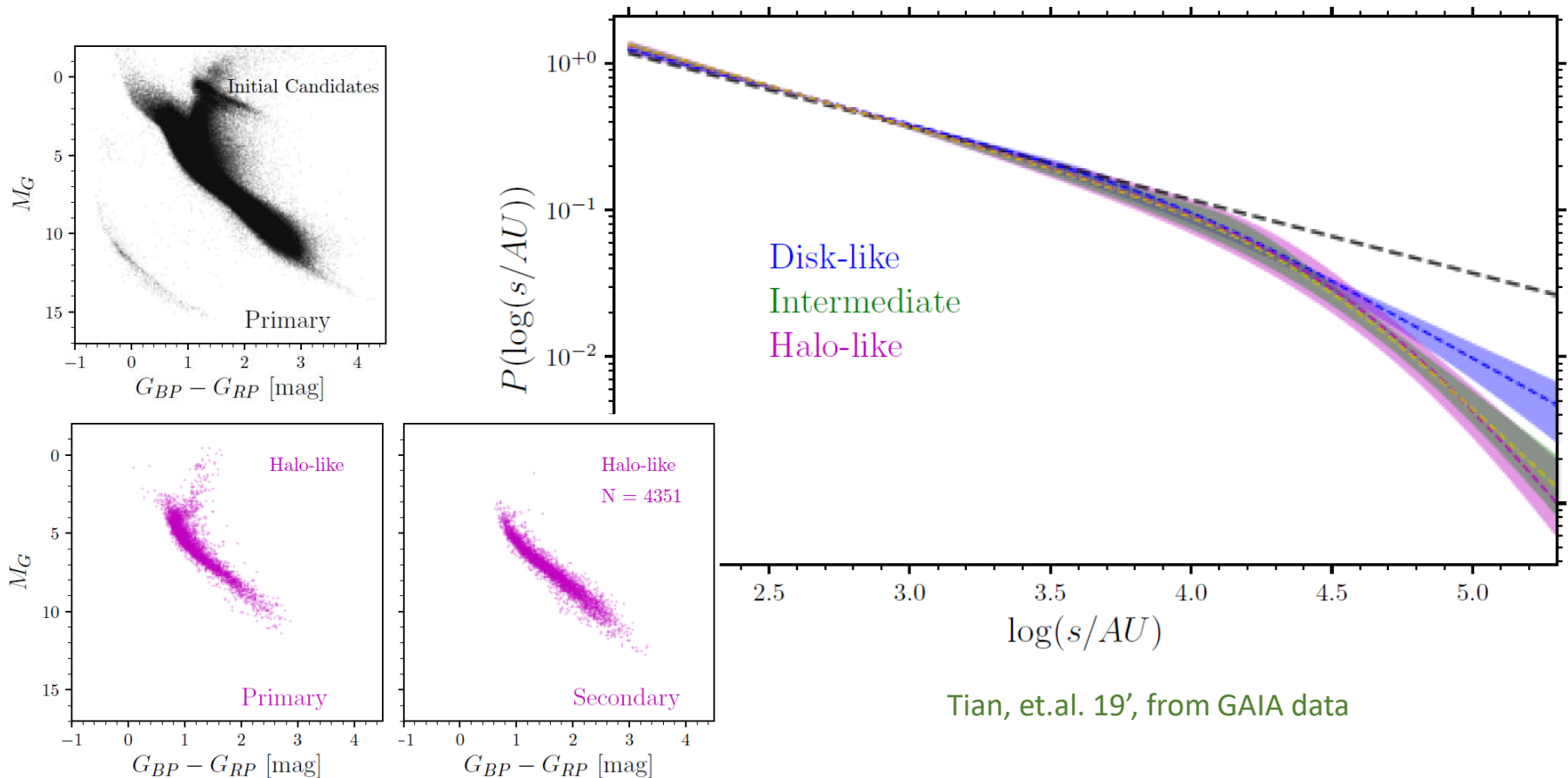


Ultra-faint dwarves (Segue I, II)

[Dalal & Kravtsov, 2203.05750](#), quote: ... derive a lower limit on the dark matter particle mass of $m_{\text{FDM}} > 3 \cdot 10^{-19} \text{ eV}$ at 99% C.L.[via simulated stellar motion]

Smaller ($\ll$ kpc) objects for small structures?

Our Galaxy hosts a population of very wide (up to ~ 1 pc) binaries/candidates



Evaporation: increment of energy leads to final disruption of the binary system.

For binary evaporation, the two-body system can be reduced to a well-defined single-body problem (plus some input from its COM motion)

$$\Delta E = \mu \vec{v}_r \cdot \Delta \vec{v}_r + \frac{1}{2} \mu (\Delta \vec{v}_r)^2$$

$\nabla \Phi$ drives the CM's random walk (can be small compared to $v_{cm} \sim 10^{-3} c$)

Tidal acceleration should be in proportional to $\partial \wedge \partial \Phi$

$$\Delta \vec{v}_r = \Delta \vec{v}_1 - \Delta \vec{v}_2,$$

$$\frac{\langle \Delta \vec{v}_r^2 \rangle}{T} = \frac{1}{T} (\langle \Delta \vec{v}_1^2 \rangle + \langle \Delta \vec{v}_2^2 \rangle - 2 \langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle)$$

$$\frac{\langle \Delta E \rangle}{T} = \mu \frac{\vec{v}_r \cdot \langle \Delta \vec{v}_r \rangle}{T} + \frac{1}{2} \mu \left(\frac{\langle \Delta \vec{v}_1^2 \rangle}{T} + \frac{\langle \Delta \vec{v}_2^2 \rangle}{T} - \frac{2 \langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle}{T} \right)$$

$\langle \rangle$: spectral averaging over the soliton ensemble.

$1/T$: time averages... should also take account of the binary's Keplerian motion.

Slow orbit: wide binaries

$$\frac{\lambda_{\text{DM}}}{v} \ll T \ll \frac{2\pi}{\omega_b}$$

For wide binaries, $v_r / v_{\text{cm}} \sim 10^{-3}$. For a longer orbit period than the time scale of potential variations, one can take the ensemble average first, and leave the orbital parameters as constants.

Velocity change driven by the potential's gradient

$$\Delta \vec{v} = -i \int_0^T dt \int \frac{\vec{k} d^3 k d\omega}{(2\pi)^4} \Phi(\vec{k}, \omega) e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

To include the gravitational potential variations, expand out $\vec{r}(t)$ that travels across the DM background

$$\vec{r}(t) \approx \vec{r}_0 + \vec{v}_0 t + \int_0^t ds (t-s) \dot{\vec{v}}(s)$$

$$\begin{aligned} \exp \left[i(\vec{k} \cdot \vec{r} - \omega t) \right] &= \exp \left[i\vec{k} \cdot \left(\vec{r}_0 + \vec{v}_0 t + \int_0^t d\tau (t-\tau) \dot{\vec{v}}(\vec{r}_0 + \vec{v}_0 \tau, \tau) \right) - i\omega t \right] \\ &\approx e^{i\vec{k} \cdot (\vec{r}_0 + \vec{v}_0 t) - i\omega t} \left[1 + i\vec{k} \cdot \int_0^t d\tau (t-\tau) \dot{\vec{v}}(\vec{r}_0 + \vec{v}_0 \tau, \tau) \right] \end{aligned}$$

The first term will vanish after the ensemble average.
The only contribution comes from the second term

Expand out the equation

$$\Delta \vec{v} = i \int_0^T dt \int_0^t d\tau (t - \tau) \int \frac{\vec{k} d^3 k d\omega}{(2\pi)^4} \int \frac{(\vec{k} \cdot \vec{k}') d^3 k' d\omega'}{(2\pi)^4} \Phi(\vec{k}, \omega) \Phi^*(\vec{k}', \omega') e^{i\vec{k} \cdot (\vec{r}_0 + \vec{v}_0 t) - i\omega t} e^{-i\vec{k}' \cdot (\vec{r}_0 + \vec{v}_0 \tau) + i\omega' \tau}$$

And make use of the relation:

$$\langle \Phi(\vec{k}, \omega) \Phi^*(\vec{k}', \omega') \rangle = (2\pi)^4 C_\Phi(\vec{k}, \omega) \delta^3(\vec{k} - \vec{k}') \delta(\omega - \omega')$$

We can make it an integral over the correlation function.

$$\langle \Delta \vec{v} \rangle = \int_0^T dt \int_0^t d\tau \int \frac{d^3 k d\omega}{(2\pi)^4} \vec{k}^2 C_\Phi(\vec{k}, \omega) \frac{\partial}{\partial \vec{v}_0} e^{i(\vec{k} \cdot \vec{v}_0 - \omega)(t - \tau)}$$

The 1st order 'diffusion' coefficient:

$$D[\Delta \vec{v}] = \frac{\langle \Delta \vec{v} \rangle}{T} = -\frac{1}{2} \int \frac{\vec{k} d^3 k d\omega}{(2\pi)^3} \vec{k}^2 C_\Phi(\vec{k}, \omega) K'_T(\omega - \vec{k} \cdot \vec{v}_0)$$

where $K'_T(\omega) = \frac{\omega T \sin(\omega T) - 2[1 - \cos(\omega T)]}{\pi \omega^3 T}$

Correlation functions

Spectra of the two-point correlation functions for the density ρ and gravitational potential Φ

$$\langle \rho(\vec{r}, t) \rho(\vec{r}', t') \rangle \equiv C_\rho(\vec{r} - \vec{r}', t - t')$$

$$\langle \Phi(\vec{r}, t) \Phi(\vec{r}', t') \rangle \equiv C_\Phi(\vec{r} - \vec{r}', t - t')$$

Fourier trans.

$$C_\rho(\vec{r}, t) = \int \frac{d^3k d\omega}{(2\pi)^4} C_\rho(\vec{k}, \omega) e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

By Poisson Eq. $\nabla^2 \Phi = 4\pi G \rho$

$$C_\Phi = 16\pi^2 G^2 k^{-4} C_\rho$$

$$C_\rho(\vec{k}, \omega) = \frac{1}{m_s} \int d^3r d^3r' d^3v dt \rho(\vec{r}) \rho(\vec{r}') F(\vec{v}) e^{-i\vec{k} \cdot (\vec{r} + \vec{r}' + \vec{v}t)} e^{i\omega t}$$

$$F(\vec{v}) = \frac{\rho_0}{(2\pi\sigma^2)^{\frac{3}{2}}} e^{-\frac{v^2}{2\sigma^2}}$$

Small-sized solitons are more likely virialized so we assume a Maxwellian velocity distribution.



Performing an average over their v -distribution



$\rho(k)$ is the F.T. of the soliton profile

$$C_\rho(\vec{k}, \omega) = \frac{1}{m_s} \rho^2(\vec{k}) \rho_0 \sqrt{\frac{2\pi}{k^2 \sigma^2}} e^{-\frac{\omega^2}{2k^2 \sigma^2}}$$

Quadratic terms dominate kinetic energy growth.

$$\frac{1}{2}\mu \left(\frac{\langle \Delta \vec{v}_1^2 \rangle}{T} + \frac{\langle \Delta \vec{v}_2^2 \rangle}{T} - \frac{2\langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle}{T} \right)$$

An accurate and concise form factor is obtained after lengthy algebra

$$\frac{\langle \Delta \vec{v}_1^2 \rangle}{T} = \int \frac{\vec{k}^2 d^3 k}{(2\pi)^3} C_\Phi(\vec{k}, \vec{k} \cdot \vec{v}_1)$$

$$\frac{\langle \Delta \vec{v}_2^2 \rangle}{T} = \int \frac{\vec{k}^2 d^3 k}{(2\pi)^3} C_\Phi(\vec{k}, \vec{k} \cdot \vec{v}_2)$$

[Use approximation]

$$v_c \gg v_r, v_1 \approx v_2 \approx v_c$$



$$\frac{\langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle}{T} = \int \frac{\vec{k}^2 d^3 k}{(2\pi)^3} C_\Phi(\vec{k}, \vec{k} \cdot \vec{v}_c) \cos[\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)]$$

Finally, the tidal heating rate:

Remember $|\vec{r}_1 - \vec{r}_2| \sim 2a$, soliton size becomes relevant.

$$\frac{\langle \Delta E \rangle}{T} = \sqrt{\frac{2}{\pi}} \frac{\mu \rho_0 G^2}{m_s \sigma} \int \frac{d^3 k}{k^3} \rho^2(\vec{k}) e^{-\frac{(\vec{k} \cdot \vec{v}_c)^2}{2k^2 \sigma^2}} \left[2 \left(1 - \cos \left[\vec{k} \cdot (\vec{r}_1 - \vec{r}_2) \right] \right) \right]$$

This convolution factor takes care of scale dependence and reveals the difference btw the 2-body relative motion and the Brownian walk of a single-point particle (COM) under tidal perturbations.

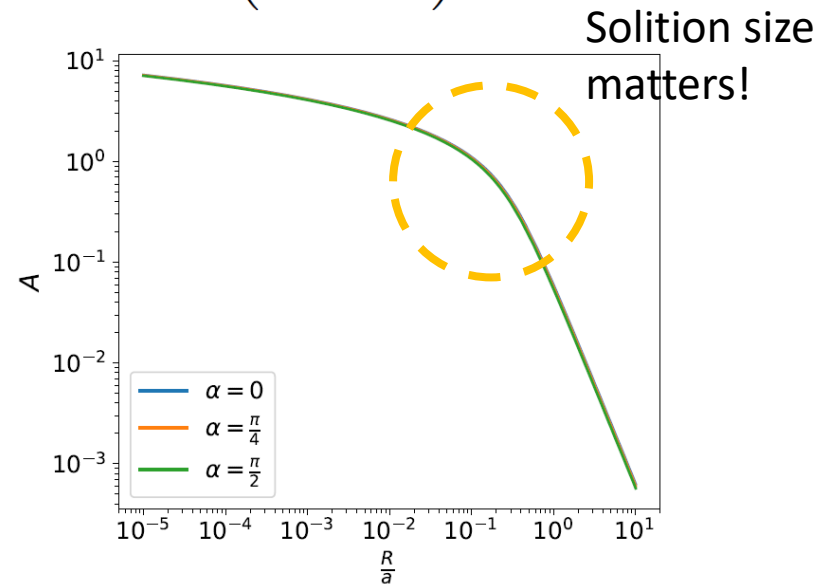
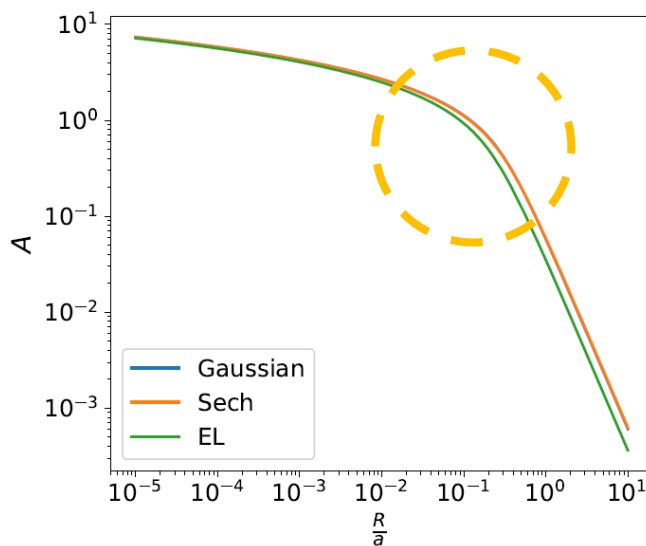


Size effect in evaporation time scale

$$\frac{\langle \Delta E \rangle}{T} = \frac{8\pi\mu\rho_0 G^2 m_s}{v_c} A \left(r_x, r_y, r_z, R, \frac{v_c}{\sigma} \right)$$

Re-define our integrals and make a similar-looking to the classical formula

The relaxation time:
$$t_d = \frac{|E_0|}{\left(\frac{dE}{dt}\right)_0} \int_0^1 \frac{dx}{A \left(\frac{R}{a_0} x, \frac{v_c}{\sigma} \right)}$$



$$A = \frac{1}{\sqrt{2\pi}} \frac{v_c}{\sigma} \int_0^{+\infty} \frac{dk}{k} \frac{\rho^2(k)}{m_s^2} \int_{-1}^1 dx e^{-\frac{v_c^2 x^2}{2\sigma^2}} \left[1 - J_0 \left(k \sqrt{r_x^2 + r_y^2} \sqrt{1 - x^2} \right) \cos(k r_z x) \right]$$

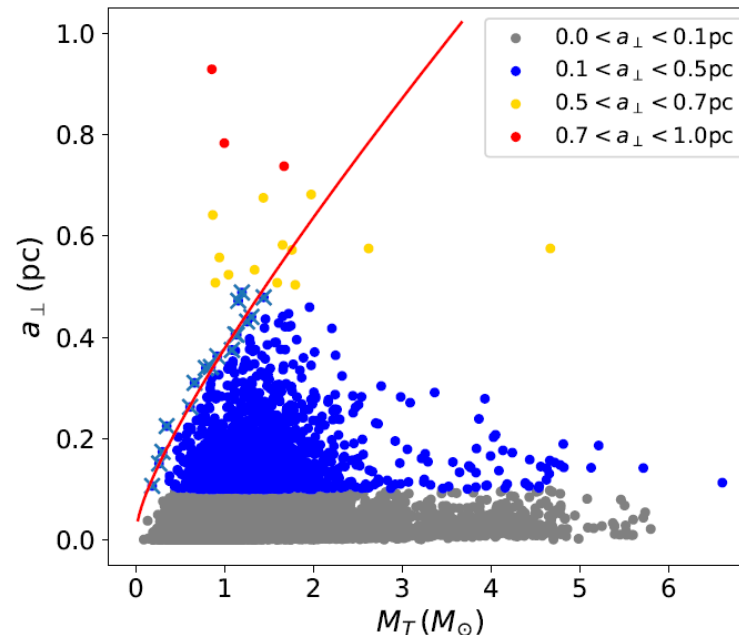
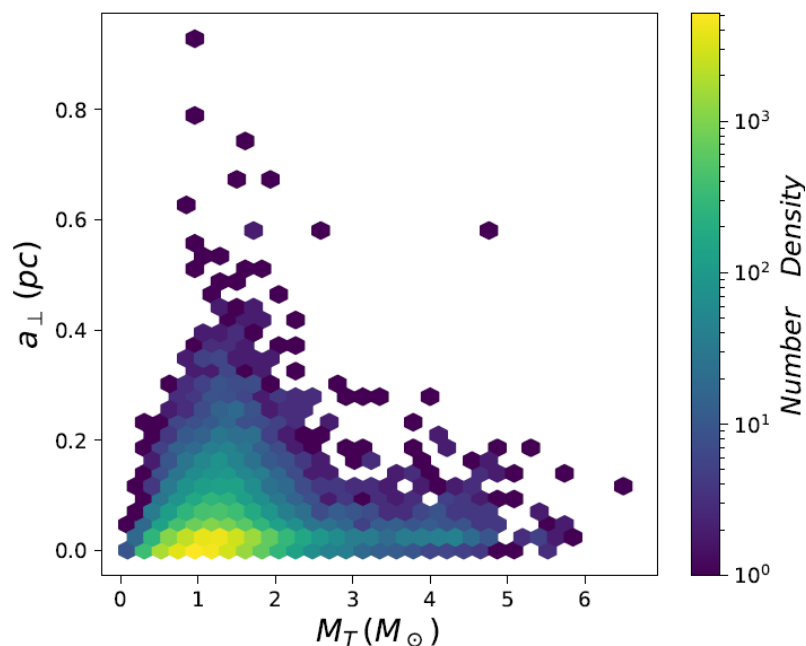
GAIA catalog: halo-like wide binaries

- Select old binary candidates with large tangential velocity
- Select candidates with a low probability of being aligned by chance
- Veto candidates with close companions, many neighbors ($N < 2$)
- Exclude candidates containing white dwarves

El-Badry and H.-W. Rix, Mon.
Not. Roy. Astron. Soc. 480,
4884 (2018)

2000+ candidates pass selection cuts out of 62990 from GAIA data

Gaia Collaboration, Astronomy & Astrophysics 649, A9 (2021), 2012.02036.



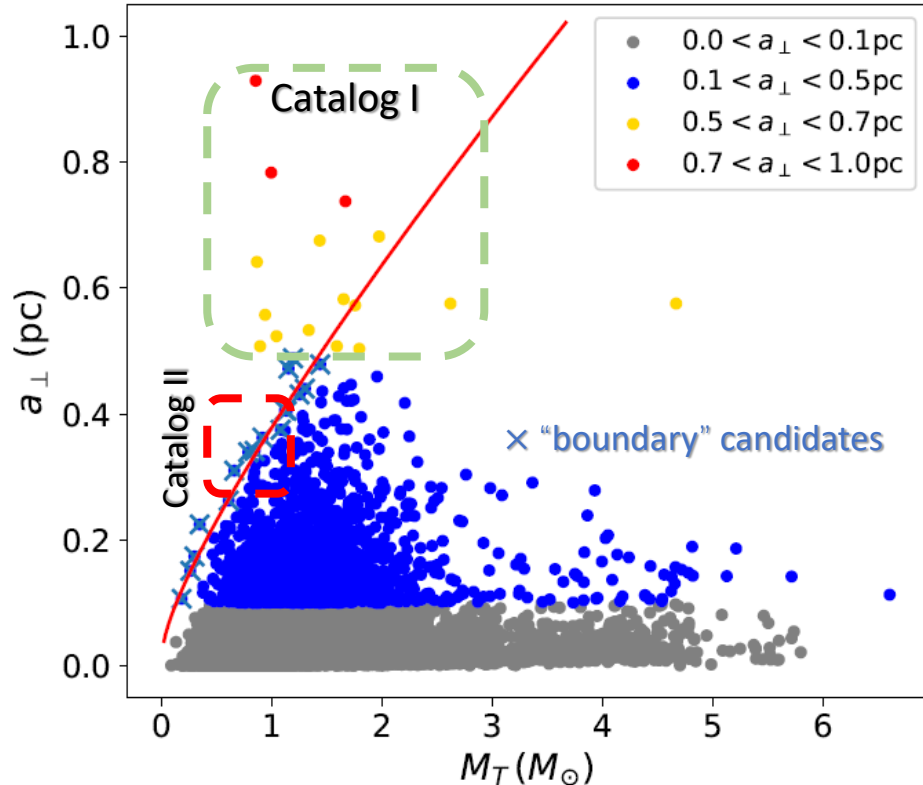
Select high-prob. candidates with large separation and relatively lower mass.

Require their average evaporation to be longer than 10 Gyr

$$\langle t_d \rangle \equiv \frac{1}{N} \sum_i t_{d,i} < 10 \text{ Gyr},$$

Source id1	Source id2	parallax1	parallax2	g mag1	g mag2	R chance align	M_1	M_2	M_T	$a_\perp$ (pc)
1312689344512158848	1312737894822499968	3.375	3.310	12.07	17.21	0.000996	0.950	0.483	1.432	0.675
6644959785879883776	6644776515331203840	2.007	2.354	17.85	18.00	0.0462	0.440	0.412	0.851	0.929
2305945096292235648	2305945538674043392	2.366	2.316	15.74	17.30	1.53e-09	0.518	0.373	0.891	0.508
2127864001174217088	2127863726296352256	1.370	1.363	13.64	15.60	0.0357	0.924	0.741	1.665	0.737
577970351704355072	580975626220823296	3.117	3.021	16.35	17.47	0.0850	0.484	0.452	0.937	0.557
1401312283813377536	1401310698969746944	1.244	1.234	16.97	18.92	0.0113	0.631	0.409	1.040	0.523
1559537092292382720	1559533965556190848	1.209	1.224	13.63	15.03	0.00142	1.117	0.854	1.971	0.682
5476416420063651840	5476421406528047104	1.204	1.214	13.66	15.46	0.0834	1.016	0.775	1.791	0.503
4004141698745047040	4004029857796571136	5.104	5.100	14.09	16.07	0.00492	0.580	0.412	0.992	0.783
6779722291827283456	6779724009814201984	1.575	1.579	17.72	18.79	0.00712	0.484	0.378	0.862	0.641
3594791561220458496	3594797539814936832	1.065	1.069	14.44	16.31	0.0763	0.917	0.731	1.649	0.582
3871814958946253312	3871818601078520192	1.449	1.499	15.66	17.13	0.0188	0.676	0.657	1.333	0.533
2379971950014879360	2379995177198014976	1.604	1.588	14.21	16.58	0.0270	0.876	0.712	1.588	0.507
6826022069340212864	6826040868412655872	2.379	2.373	11.76	14.05	0.0987	1.016	0.738	1.754	0.572
5798275535462480768	5798276325736369024	1.247	1.261	13.32	13.76	0.000536	1.443	1.176	2.619	0.575

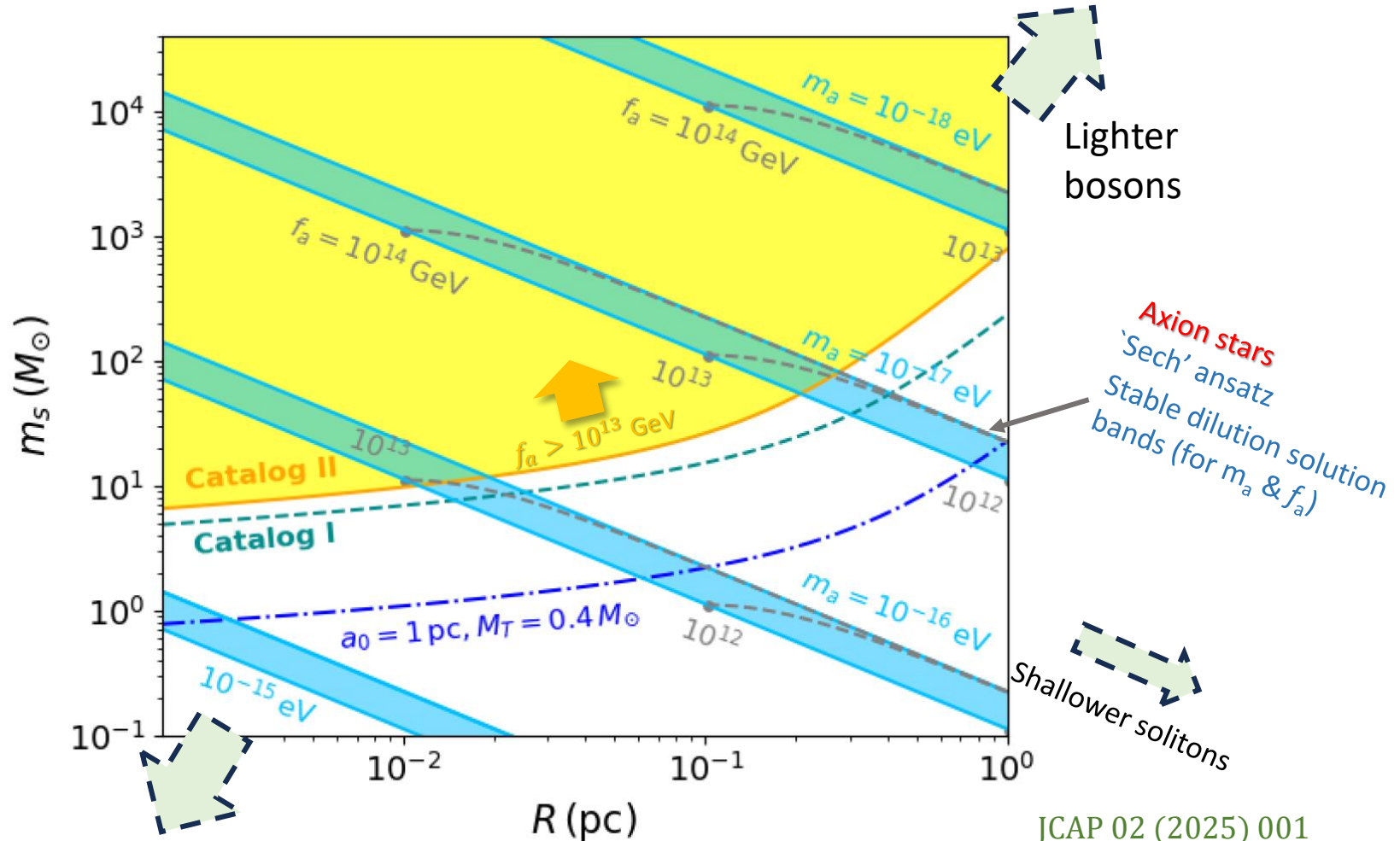
TABLE I: (Catalog I). High probability halo-like wide binaries with $a_\perp > 0.5$ pc and $M_T < 3 M_\odot$.



- Binaries shift left-ward over time under tidal perturbations
- A $t_d=10$ Gyr curve and its soliton configuration (m_s , R) is *disfavored* if lots of candidates reside on its left/top side
- (red line) on the boundary of populated region corresponds to $t_d = 10$ Gyr with ($m_s=9.3m_\odot$, $R=0.03$ pc)

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Catalog I & II limits for $t_d = 10^{10}$ yr under solitons' tidal perturbation.
 (Blue bands) solutions for ALPs with potential: $V(a) \sim -m_a^2 f_a^2 \cos(a/f_a)$



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Heavier bosons

Brief notes about axion stars

Derive the non-relativistic Hamiltonian and find a minimum.

Real scalar Lagrangian: $\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \nabla_\mu a \nabla_\nu a - V(a) \right]$

Nonrelativistic approximation: $a(\vec{x}, t) = \frac{1}{\sqrt{2m_a}} \left(e^{-im_a t} \psi(\vec{x}, t) + e^{im_a t} \psi^*(\vec{x}, t) \right)$

$$n(\mathbf{x}) = \psi^*(\mathbf{x})\psi(\mathbf{x}) \text{ and } \rho(\mathbf{x}) = m \psi^*(\mathbf{x})\psi(\mathbf{x})$$

Nonrelativistic Lagrangian, with $g_{00} = 1 + 2\phi_N$:

$$\mathcal{L}_{nr} = \frac{i}{2} \left(\dot{\psi} \psi^* - \psi \dot{\psi}^* \right) - \frac{1}{2m} \nabla \psi^* \cdot \nabla \psi - V_{nr}(\psi, \psi^*) - m \psi^* \psi \phi_N(\psi^*, \psi)$$

Stable solution: presence of a minimum
of the reduced Hamiltonian.

Use dimensionless
variables:

$$\begin{aligned}\tilde{H} &= \frac{\sqrt{|\lambda|^3}}{m_a^2 \sqrt{G}} H, \\ \tilde{N} &= m_a \sqrt{G|\lambda|} N, \\ \tilde{R} &= m_a^2 \sqrt{\frac{G}{|\lambda|}} R,\end{aligned}$$

Rescaled energy:

$$\tilde{H} = a \frac{\tilde{N}}{\tilde{R}^2} - b \frac{\tilde{N}^2}{\tilde{R}} - c \frac{\tilde{N}^2}{\tilde{R}^3}$$

Plug into ansatz:

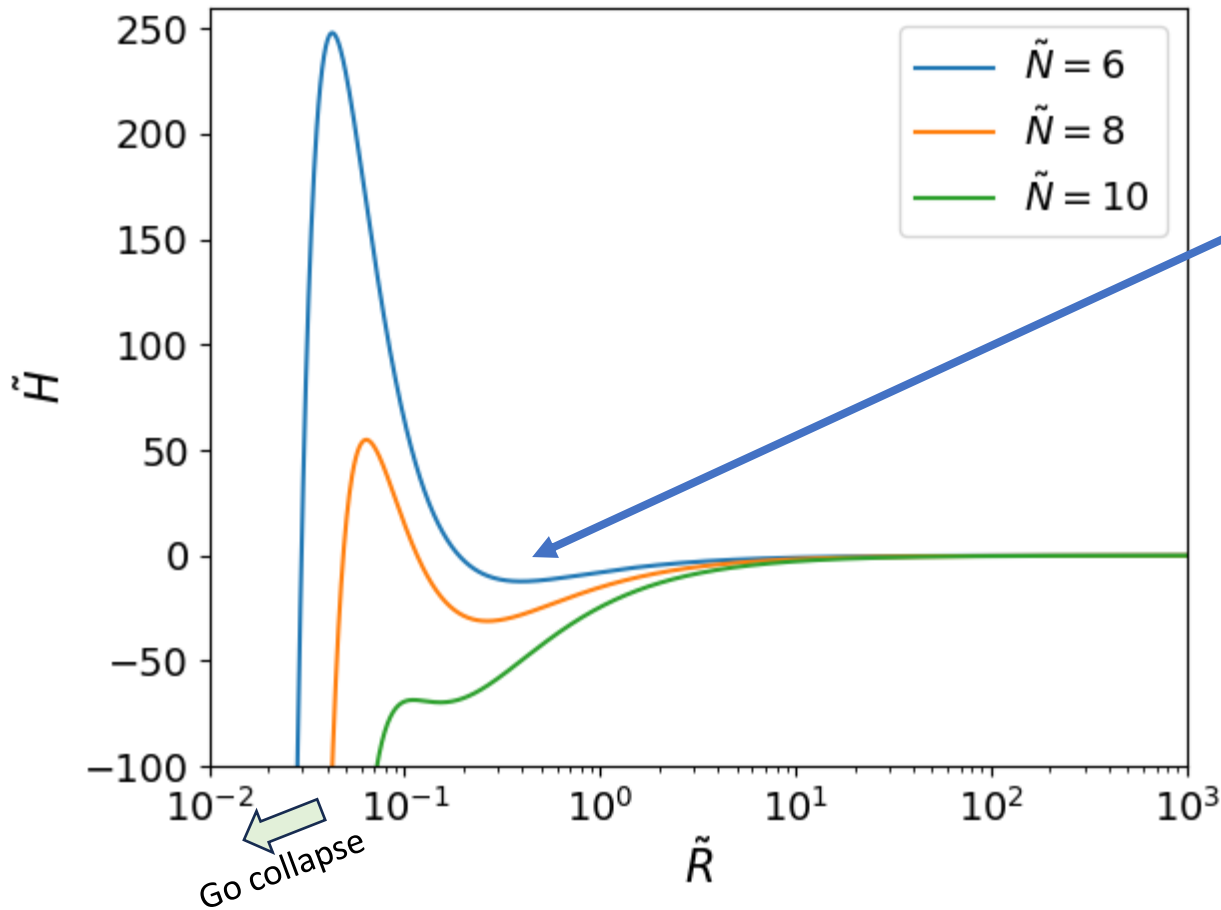
$$a = \frac{12 + \pi^2}{6\pi^2}, \quad b = \frac{6 [12\zeta(3) - \pi^2]}{\pi^4}, \quad c = \frac{\pi^2 - 6}{8\pi^5}$$

At low density, the axion
number stays unchanged.
For fixed $\tilde{N}$, $\tilde{H}$ can
minimize at a certain radius

$$\tilde{R} = \frac{a \pm \sqrt{a^2 - 3bc\tilde{N}^2}}{b\tilde{N}}$$

Growth beyond this point
destabilizes the axion star.

Solution stability at fixed $\tilde{N}$



Presence of a local minimum: **stable** solution

Stable dilute axion star solution exists if

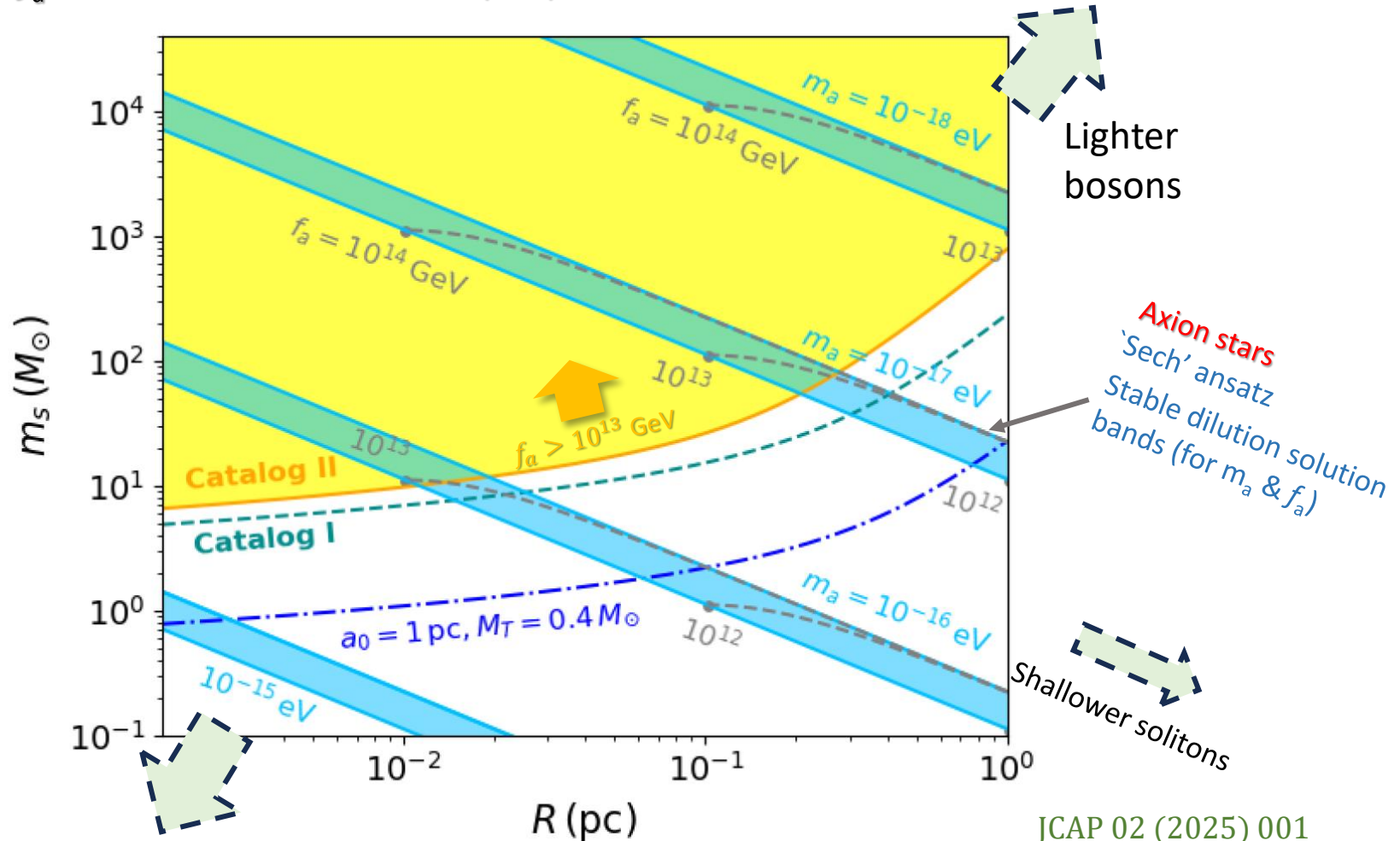
$$\tilde{N} \leq 10.12$$

For larger axion star mass, it will collapse to form a **dense axion star**

(Blue bands) show stable dilute-star solutions

for ALP potential: $V(a) \sim -m_a^2 f_a^2 \cos\left(\frac{a}{f_a}\right)$

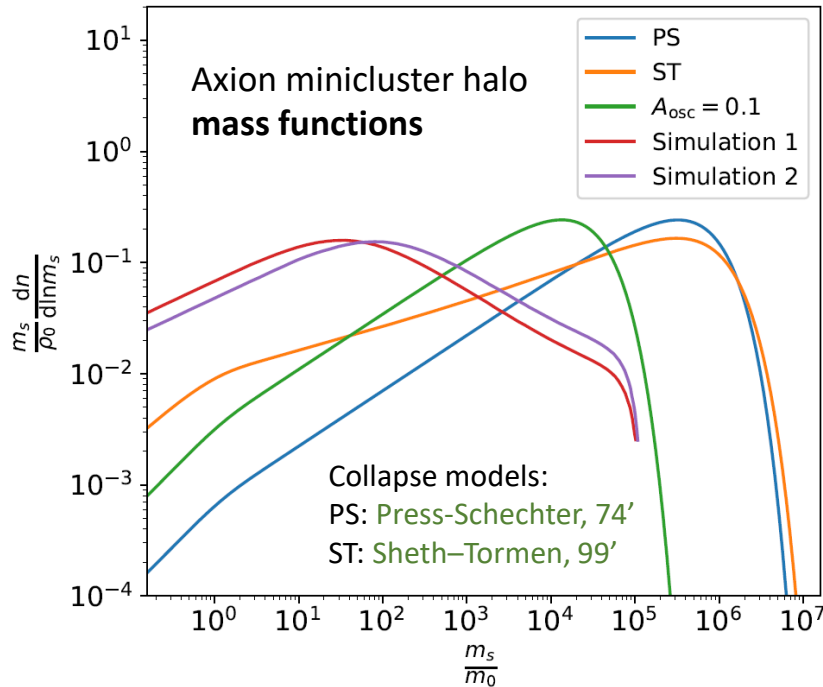
Larger f_a reaches denser solutions (left).



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Binary disruption bounds: for ALP minicluster halos

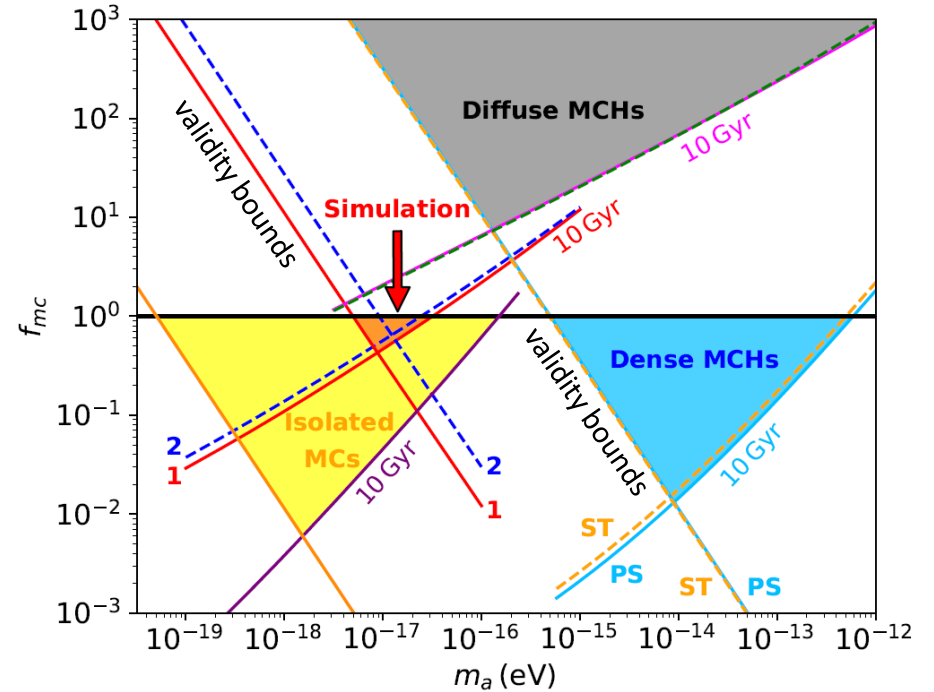
More 'realistic' axion distributions from
numeric and semi-analytic calculations



$$m_0 = 281 M_{\odot} \left(\frac{m_a}{10^{-16} \text{ eV}} \right)^{-\frac{3}{2}} \frac{g_{*S}(T_{\text{osc}})}{g_{*S}(T_0)} g_*^{-\frac{3}{4}}(T_{\text{osc}})$$

Denser models of MCH indicate
for more significant disruption effect.

Phys.Rev.D 111 (2025) 4, 043042

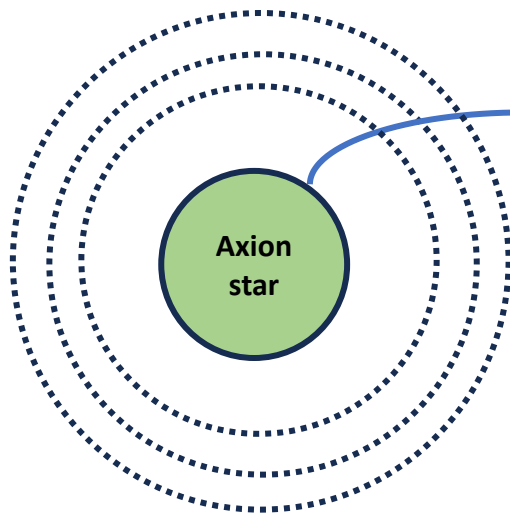


Condensation (soliton size) models,

$$\rho(r) = \frac{\rho_c}{\frac{r}{r_s}} \left(1 + \frac{r}{r_s} \right)^{-2} \cdot \frac{200}{3} \frac{c^3}{\ln(1+c) - \frac{c}{1+c}}$$

Diffuse: low concentration, $c \sim 10$, larger MC size
Dense: high concentration, $c \sim 10^4$, compact MCs
Isolated: MCs do not clump into MCHs

Axion star forms inside a 'minicluster' and feeds on its host.



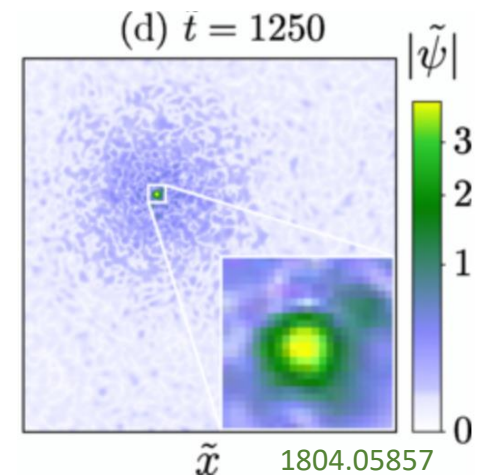
Fuzzy axion halo
(minicluster)
has a nonzero velocity

Axion star: large number of axions
stay on the same (BEC) state ψ_S

'BEC in virialized DM halos'
see [Levkov, Panin, Tkachev, 18'](#),
formation under self-gravity



Axion star will further
grow and potential trigger
run-away self-interaction
(instability).



Star growth calculation:
halo axion capture by
BEC via self-gravity.
see [Chan, Sibiryakov, Xue, 24'](#)



Potential signals?

Q1: Instability requires
the incorporation of
axion's self-interaction:
dilute stars :gravity and
self-interaction
equilibrium.

Q2: Mapping growth
stability into axion
parameters space (m_a, f_a)
requires incorporation of
the self-interaction
potential

A quantum calculation for axion capture $\psi_g + \psi_g \rightarrow \psi_s + \psi_g$
via self-interaction potential $V \sim a^4$

$$S = \int d^4x \left(\frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 - \frac{1}{24} \lambda a^4 \right)$$

$S_g =$  Nonrelativistic limit and take ψ_s as a bkg state

$$\int dt d^3\vec{x} \left(i\psi_g^* \partial_t \psi_g + \frac{1}{2m_a} \psi_g^* \nabla^2 \psi_g - \frac{\lambda}{16m_a^2} |\psi_g + \psi_s|^4 \right)$$

$$|\mathcal{M}_{g+g \rightarrow g+s}|^2 = \left| \int d^3x dt \langle f | \mathcal{H}_{\text{int}} | i \rangle \right|^2 = \frac{\lambda^2}{64m_a^4} \left| \int d^3x dt \langle f | \psi_s^* |\psi_g|^2 \psi_g | i \rangle \right|^2$$

ψ_s : BEC's common soliton state

$$\psi_s(\vec{x}, t) = \chi(r) e^{-i\epsilon_s t}$$

$\chi(r)$ takes stable solution ansatzes.

ψ_g : 'gas' state in virialized minicluster

$$\psi_g(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} a_{\vec{k}} \varphi_{\vec{k}}(\vec{x}) e^{-i\epsilon_{\vec{k}} t}$$

$$\varphi_{\vec{k}}(\vec{\xi}) = e^{i\vec{k} \cdot \vec{\xi}} \Gamma\left(1 - \frac{i\beta}{\kappa}\right) e^{\frac{\pi\beta}{2\kappa}} {}_1F_1\left[\frac{i\beta}{\kappa}, 1, i(\kappa\xi - \vec{k} \cdot \vec{\xi})\right]$$

Scattering states under Coulomb potential

Amp calculation includes both axion
self-interaction and self-gravity, [2508.14535](#)

Typical horizon collapse minicluster mass (host mass, M_0)

$$M_0 = \frac{4\pi^4}{3} \frac{\bar{\rho}_a(t_0) R_{\text{osc}}^{-3}}{H^3(t_{\text{osc}})} = 1.235 \times 10^{-9} M_\odot \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-0.561}$$

Axion star initial mass (by num. simulation, M_{s0}),
from [Levkov et.al. 2024](#)

$$M_{s0} = 8.384 \times 10^{-11} M_\odot \times \alpha_0 [\delta^3(1+\delta)]^{\frac{1}{6}} \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-1} \left(\frac{M_0}{M_\odot} \right)^{\frac{1}{3}}$$

Axion star max stable mass (dilute stars, M_{max})

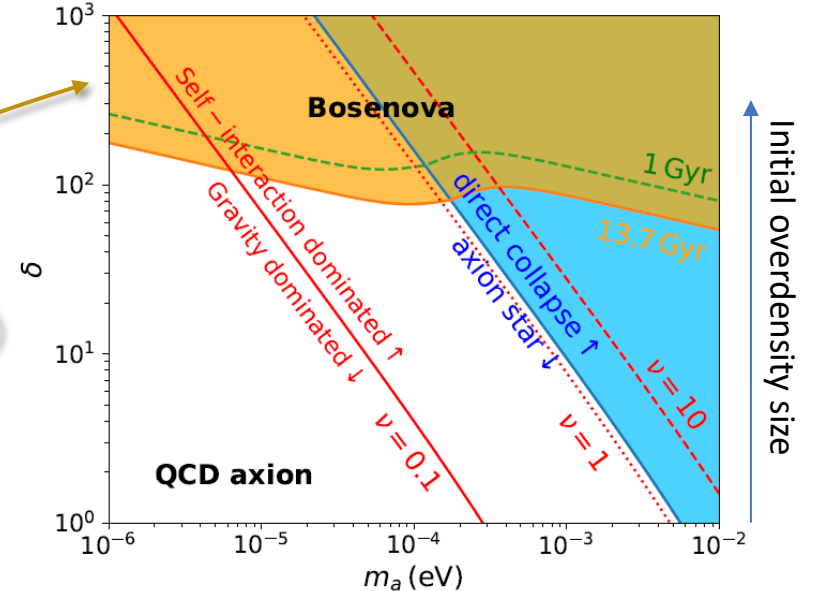
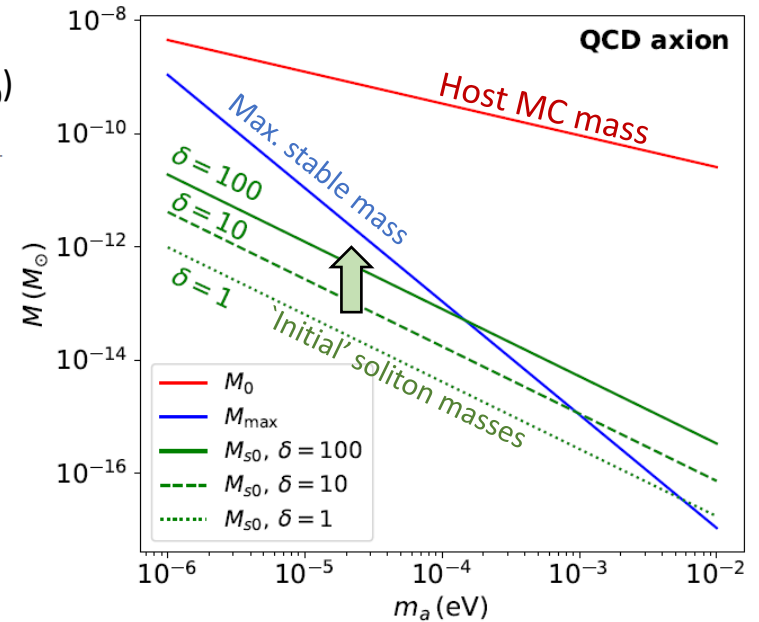
$$M_{\text{max}} = 1.089 \times 10^{-11} M_\odot \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-2}$$

Self-interaction growth time to reach M_{max}

$$t_{c,\lambda} = \frac{1}{\Gamma_\lambda} = \frac{1541 \text{ Gyr}}{[\delta^3(1+\delta)/10^8]^{\frac{5}{3}}} \times \left(\frac{\alpha_0}{\sqrt{2/5}} \right)^2 \left(\frac{\alpha(\nu, \tilde{N})}{60} \right)^{-1} \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-1.374}$$

$t_{\text{growth}} < \text{AoU}$ (yellow shaded region)

Bosenovae for QCD axion (large δ) and ALPs (quite common) in our Universe. For details see [2508.14535](#)



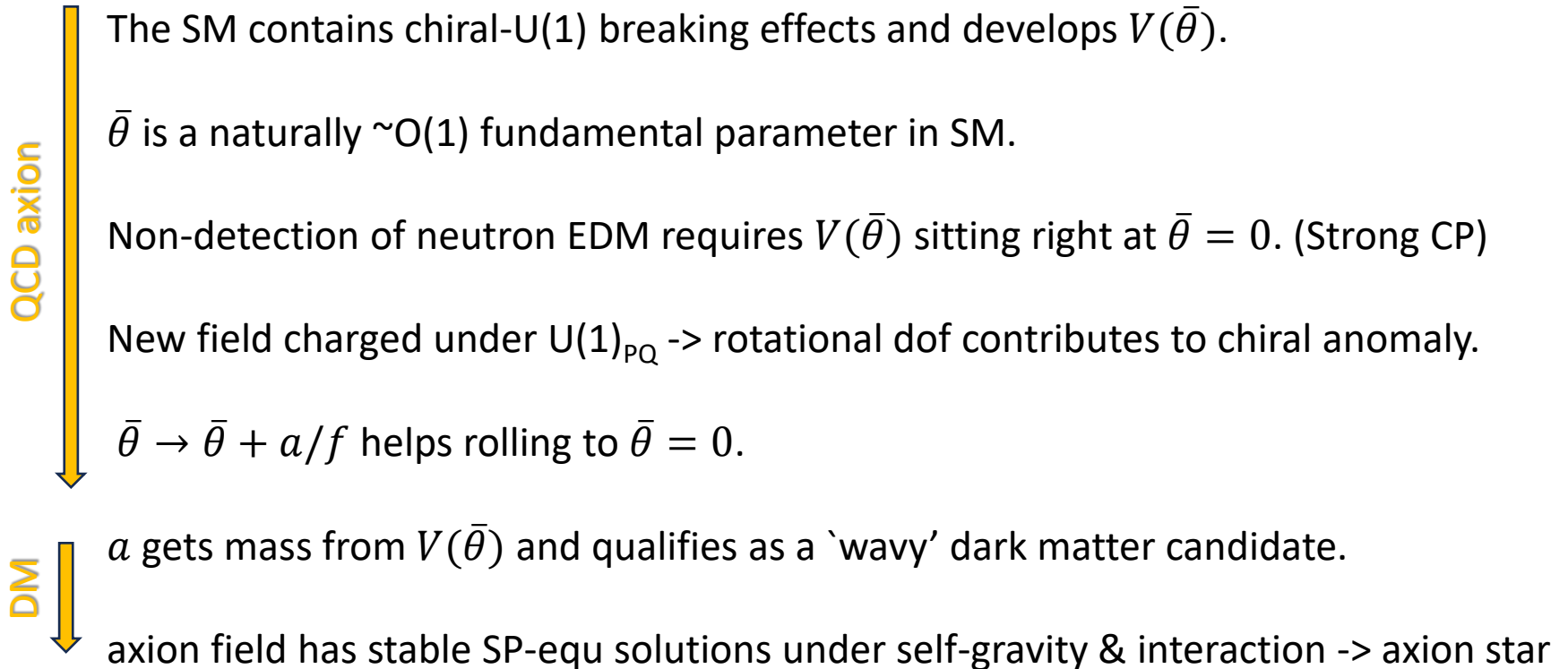
Heavier QCD axion ($\lambda = -0.346 m_a^2 f_a^2$) 24

Take home messages

- Interest in DM's gravitational imprints is on the rise.
- We provide an *analytic* form factor of binary evaporation by randomly distributed, spatially extended objects. For dilute axion stars, **sensitivity extends into heavier ALP mass range $10^{-17} < m_a < 10^{-15}$ eV.**
- GAIA data contain some *2000+ high-probability, halo-like, $a > 0.1$ pc candidates* → sensitivity to DM tidal disruptions.
- Sophisticated DM mass distributions: Tidal disruption limits are relevant.
- **Axion stars go bosenova**: possible for both QCD axion/ALP dark matter.

Backups

- (Summary version of) the Axion DM story:



Axion: a story from basic symmetries

- Weak int. violates both P and CP . Lee & Yang, 56'; Wu, 57'
- CP symmetry in strong interaction? Cronin, Fitch, 64'

$$\mathcal{L} \supset -\frac{1}{4}G^2 + \frac{\theta g_s^2}{32\pi^2} G\tilde{G}$$

In the SM, CP is broken by QCD (θ_{QCD}), as super-selection rule of QCD's instanton connected vacua

$$|\theta\rangle = N \sum e^{i\theta n} |n\rangle$$

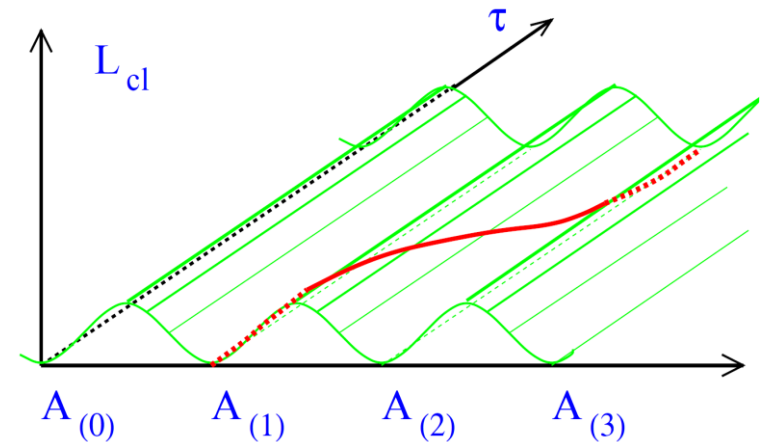
And also *explicitly broken* after $U(1)_A$ breaking in the SM.

Chiral transformation shifts θ as $e^{i\alpha Q_5} |\theta\rangle = |\theta + \alpha\rangle$

With complex quark masses, a chiral transformation on quarks add $Arg \det(M_q)$

$$\bar{\theta} = \theta_{QCD} + Arg \det M_q$$

Is the total angle, which breaks P and T (thus CP breaking), and naturally $\bar{\theta} \sim O(1)$.



Classical (PG) vacua connected by instantons, [hep-ph/0009136](https://arxiv.org/abs/hep-ph/0009136)
Vacuum min. energy @ $\theta = 0$, with Dirac spectrum assumptions (Vafa, Witten 84')

$\bar{\theta}$ is an independent parameter of the theory (SM).
It is invariant under the anomalous symmetry:

$$U \rightarrow e^{i\alpha} U, \quad \theta \rightarrow \theta - 2\alpha, \quad M \rightarrow e^{-i\alpha} M$$

$\bar{\theta}$ in-principle only determined via measurement.
'Naturally' the combined $\bar{\theta}$ should take O(1) values.

For a conceptual review, see
"Reflections on the Strong CP
Problem", R. Peccei,
[hep-ph/9807514](https://arxiv.org/abs/hep-ph/9807514)

In the effective strong interaction potential

$$\mathcal{L} = f_\pi^2 \text{Tr} \partial_\mu U \partial^\mu U^\dagger + \underbrace{a f_\pi^3 \text{Tr} MU}_{\text{breaks } U(1)_A} + b f_\pi^4 \det U + h.c.$$

Yields the minimum at

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\bar{\theta}}{2}}$$

$$\bar{\theta} = \theta + \theta_u + \theta_d$$

$$b = |b| e^{i\theta}$$

$$U = e^{i \frac{\Pi^a}{\sqrt{2} f_\pi} \sigma^a}$$

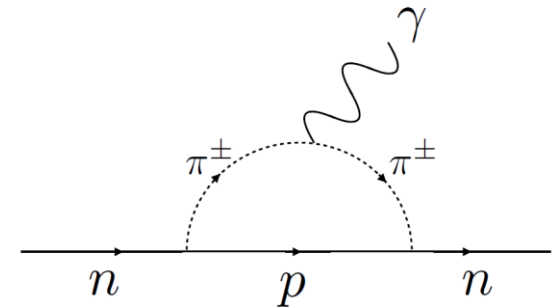
$$M = \begin{pmatrix} m_u e^{i\theta_u} & 0 \\ 0 & m_d e^{i\theta_d} \end{pmatrix}$$

The strong CP problem

$\bar{\theta}$ causes CP-violation and appears in baryon EDM:

$$\mathcal{L} = -\bar{\theta} \frac{c_+ \mu}{f_\pi} \pi^a N \tau^a N^c - i \frac{g_A m_N}{f_\pi} \pi^a N \tau^a N^c, \quad \mu = \frac{m_u m_d}{m_u + m_d}$$

$$d_n = \frac{e \bar{\theta} g_A c_+ \mu}{8 \pi^2 f_\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \sim 3 \times 10^{16} \bar{\theta} \text{ e cm}$$



Feynman diagram for neutron EDM

$$g_A \approx 1.27$$

$$c_+ \approx 1.7$$

1509.04411

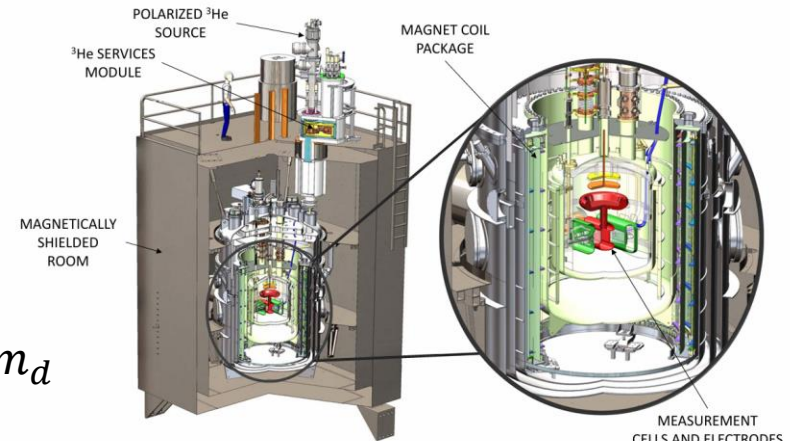
Experimental value: $\bar{\theta} \lesssim 10^{-10}$



10 orders of fine-tuning.

Some easy ways out:

- massless u -quark (G 't Hooft, 76'): $m_u < 10^{-10} m_d$
- P-parity models (Babu, Mahapatra, 90')
- Spontaneous CPV models (Nelson, 84' & Barr, 84')



nEDM @ Oak Ridge
see recent worldwide
updates: [1810.03718](#)
& Snowmass [2203.08103](#)

Peccei & Quinn's global U(1) symmetry

- After spontaneous breaking, leaves a goldstone
- Anomalous, contributes to a $U(1)_A$ rotation
- Acquires a mass (pseudo-goldstone) at low energy.

Peccei, Quinn, 77'

Assume a global $U(1)_{PQ}$ as good UV symmetry

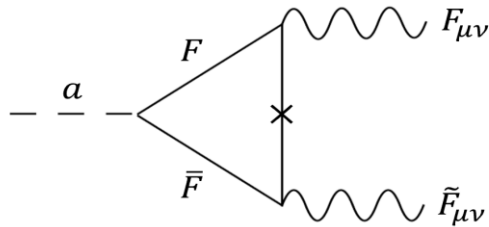
$$\begin{aligned} Q_i / U_i^c / D_i^c / L_i / E_i^c &\longrightarrow e^{i\alpha} Q_i / U_i^c / D_i^c / L_i / E_i^c , \\ H_d / H_u &\longrightarrow e^{-i2\alpha} H_d / H_u . \end{aligned} \quad (\text{as in original PQWW})$$



$U(1)_{PQ}$ breaks after 'some' scalar (has PQ charge) gets a vev $\sim O(f_a)$, leaving out a goldstone field a . The goldstone can acquire an (ABJ) anomalous coupling term :

$$\frac{a}{f_a} G\tilde{G} \quad \text{that effective 'extends' } \bar{\theta} \text{ into a dynamic field} \quad \mathcal{L} \supset \left(\frac{a}{f_a} + \theta \right) \frac{1}{32\pi^2} G\tilde{G}.$$

$$\bar{\theta} \rightarrow \bar{\theta} = \theta + \theta_u + \theta_d + \frac{a}{f_a}$$



KSVZ model (Kim-Shifman -Vainstein-Zakharov):

heavy vector-like quarks, with coupling $\lambda_Q Q^c Q S$

Axion as the Im part of an extra scalar.

The PQ U(1): $Q^c/Q \rightarrow e^{i\alpha} Q^c/Q$, $S \rightarrow e^{-i2\alpha} S$

See Kim's review: *Rev.Mod.Phys.* 91 (2019) 4, 049902 (erratum)

DSFZ model (Dine-Fischler-Srednicki-Zhitnitskii):

$$\Delta L \supset \lambda H_u H_d S^2$$

$$a = \frac{1}{\sqrt{v_u^2 + v_d^2 + v_s^2}} (v_u \text{Im} H_d + v_d \text{Im} H_u + v_s \text{Im} S)$$

$\{H_u, H_d, S\}$ charged as
 $\{-1, -1, +1\}$ under $U(1)_{\text{PQ}}$

$$f_a = \sqrt{v_u^2 + v_d^2 + v_s^2}$$

raised by large S vev.

+ Many other variants, with a central goal to increase f_a to avoid astro/flavor limits.

See review: The landscape of QCD axion models, *Phys.Rept.* 870 (2020) 1-117

Example: an effective UV construction into a Fraggett-Nielson $\left(\frac{S}{\Lambda}\right)^{n \geq 1} \bar{f}_i H f_j$ like :

A DSFZ type in case of $f \rightarrow \text{SM fermions}$, e.g. can derive from GUT/higher scale physics, & realize some flavor features.

$$\Delta L \supset -y_{ij}^u Q_i U_j^c H_u - y_{ij}^d \frac{S}{M_*} Q_i D_j^c \tilde{H}_u - y_{ij}^e \frac{S}{M_*} L_i E_j^c \tilde{H}_u$$

$$Q_i, L_i, U_i^c, D_i^c, E_i^c : 1$$

$$H_u : -2$$

$$S : -4$$

Mod.Phys.Lett.A 37 (2022) 09, 2250055 + other PQ assignments

Axion as cold dark matter

A fast oscillating field at the bottom of a $V(\phi) \sim (\phi - \phi_0)^2$ potential behaves as matter-like: $\rho(z) \sim (1+z)^3$

M. Turner, 83'

See axion cosmology review [1510.07633](#) & the more recent [2403.17697](#)

axion starts to oscillate by V_{inst} after strong QCD phase transition

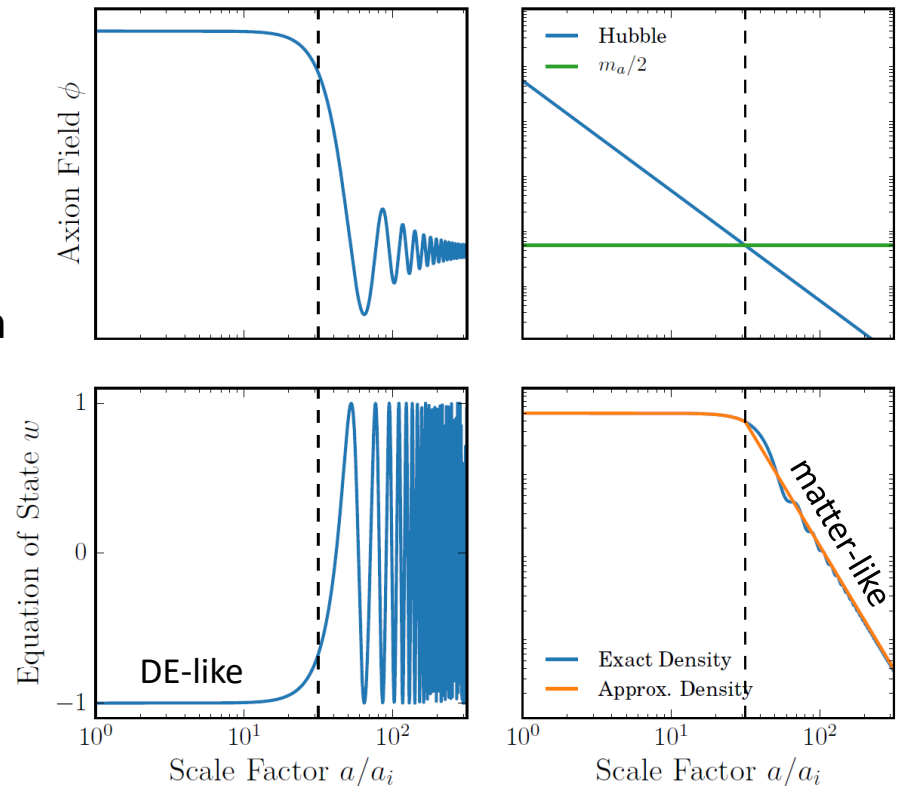
$$a(t) = a_0 \left(\frac{R_{m \sim H}}{R(t)} \right)^{2/3} \cos(m_a t)$$

Misalignment Mechanism:

axion potential overcomes Hubble friction and start oscillation from a homogeneous initial value a_0 (via inflation). Initial value gives the DM abundance:

$$\Omega_a h^2 \sim 2 \times 10^4 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{7/6} \langle \theta_{a,i}^2 \rangle$$

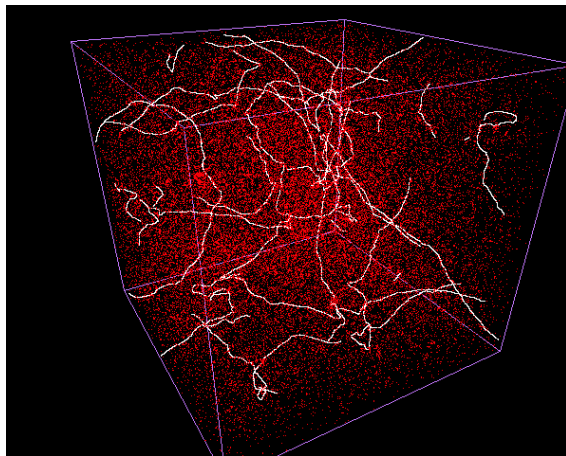
(topological defects contribute if f_a is lower than inflation scale)



Defects in post-inflation scenario

Cosmic strings form when U(1) breaks after end of inflation.

$$\phi(x) = (f_a + r(x)) e^{i\theta(x)}$$



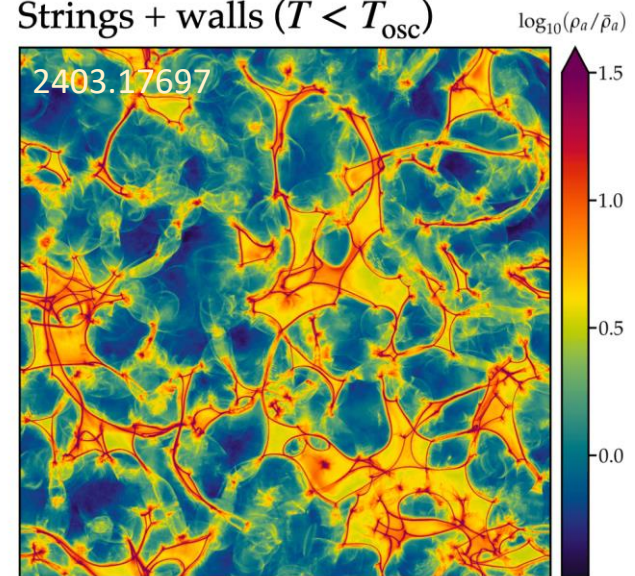
Contribute to the axion energy density:

$$\rho_a = \frac{1}{2} f_a^2 \dot{\theta}^2 + \frac{f_a^2}{2a(t)^2} (\nabla\theta)^2 + \chi(T) (1 - \cos \theta)$$

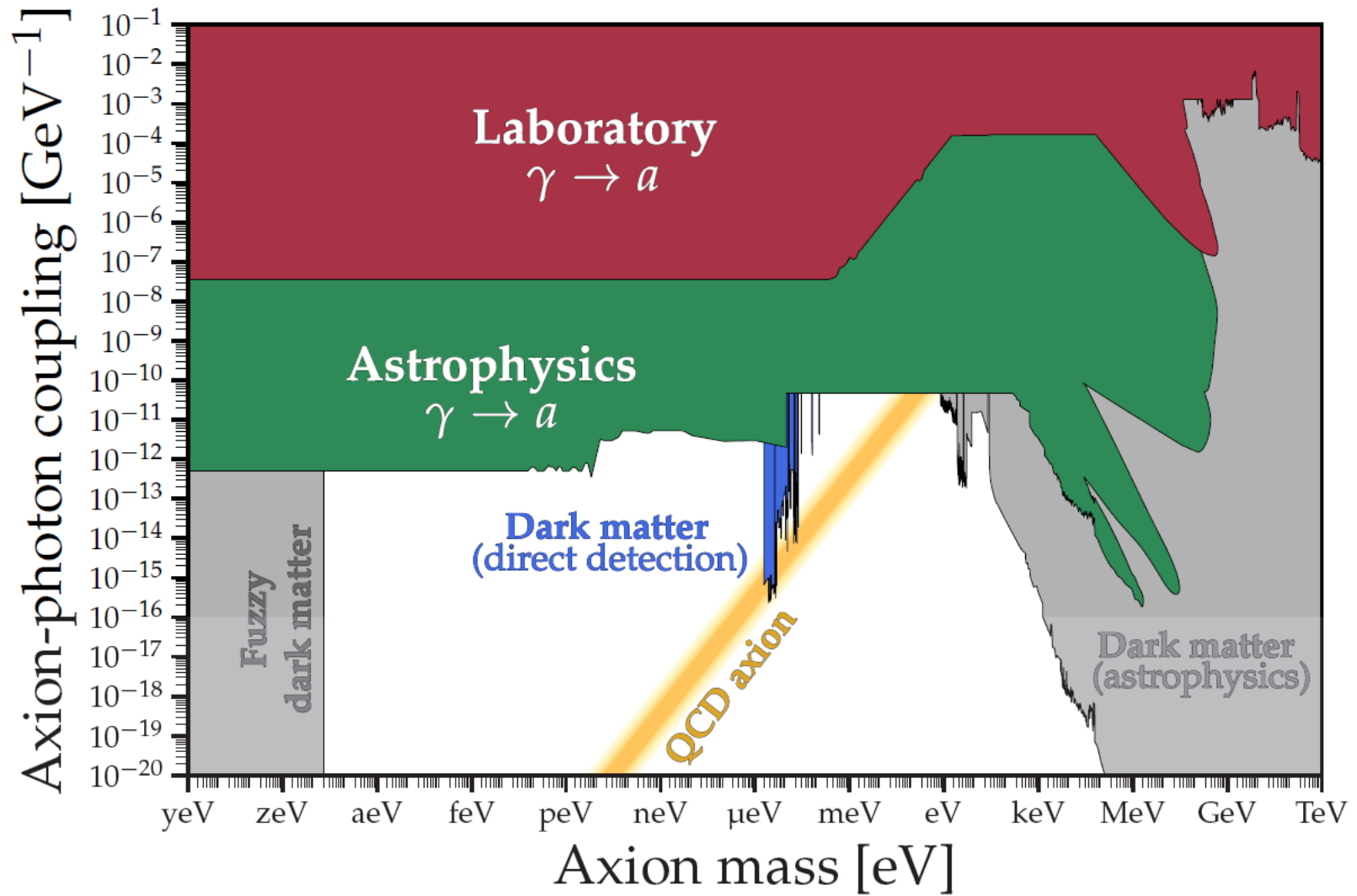
Huge amount of numerical simulation devoted to study string networks,
yet large uncertainties remain, see recent review by [Saikawa, Redondo, et.al. 2401.17253](#)

Domain walls form when V_{QCD} develops a 'true' vacuum (vacua)

Strings + walls ($T < T_{\text{osc}}$)



(DWs can unwind if $N_{\text{DW}}=1$)



Wavy Dark Matter form Objects

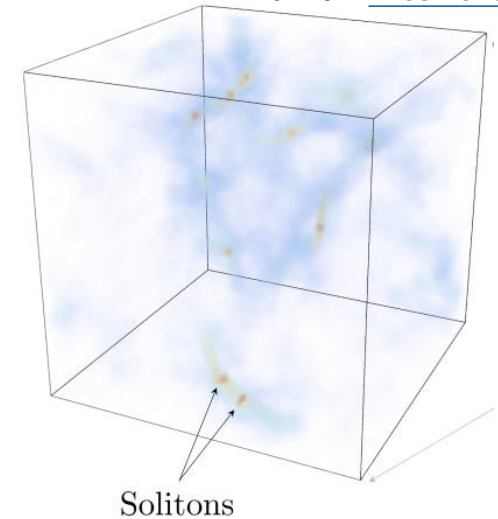
Ultra-light dark matter bosons form localized structures under gravity or self-interaction (boson stars, etc.).

The Schrodinger-Poisson (SP) equation

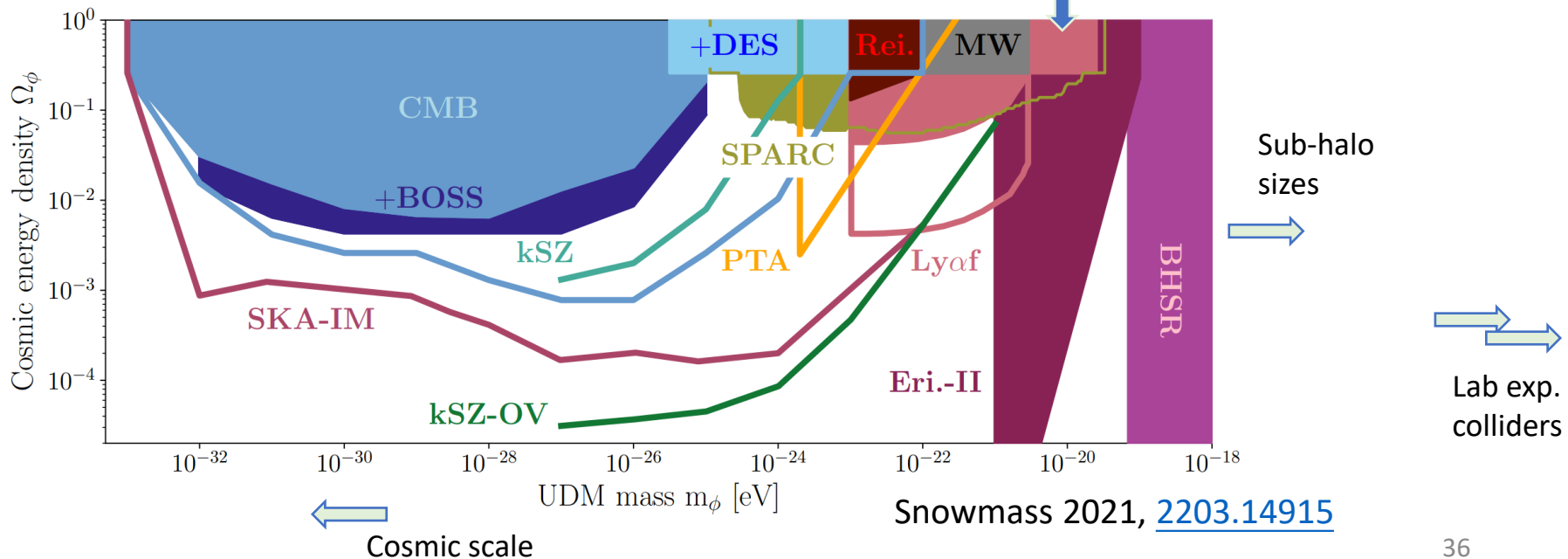
$$i\dot{\psi} = -\frac{\nabla^2\psi}{2m} - Gm^2\psi \int d^3x' \frac{\psi^*(\mathbf{x}')\psi(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{\partial}{\partial\psi^*}V_{nr}(\psi, \psi^*)$$

See Braaten & Zhang, 19' for a nice review

Pic. from [2203.10100](https://arxiv.org/abs/2203.10100)



'Fuzzy DM', Lam Hui, [1610.08297](https://arxiv.org/abs/1610.08297)



For binary evaporation, we repeat for the relative velocity $\Delta\vec{v}_r = \Delta\vec{v}_1 - \Delta\vec{v}_2$

1st term in energy increment:

$$\begin{aligned}\frac{\vec{v}_r \cdot \langle \Delta\vec{v}_r \rangle}{T} &= -\frac{1}{2} \int \frac{(\vec{k} \cdot \vec{v}_r) \vec{k}^2 d^3k d\omega}{(2\pi)^3} C_\Phi(\vec{k}, \omega) \left(K'_T(\omega - \vec{k} \cdot \vec{v}_1) - K'_T(\omega - \vec{k} \cdot \vec{v}_2) \right) \\ &= -\frac{1}{2} \int \frac{(\vec{k} \cdot \vec{v}_r) \vec{k}^2 d^3k d\omega}{(2\pi)^3} C_\Phi(\vec{k}, \omega) \left(\delta'(\omega - \vec{k} \cdot \vec{v}_1) - \delta'(\omega - \vec{k} \cdot \vec{v}_2) \right)\end{aligned}$$

For $v_1 \approx v_2 \approx v_{CM}$ and $|v_1 - v_2| \ll v_{CM}$, this contribution is suppressed by v_r / v_{CM}

The evaporation will be dominated by the 2nd order $(\Delta v)^2$ term.

Construction of the potential

An randomly distributed ensemble of solitons:

$$\rho(\vec{x}, t) = \sum_i |\varphi(\vec{x} - \vec{x}_i - \vec{v}_i t)|^2 - \langle \rho \rangle$$

Soliton profiles are typically solved from S-P equation. Sample ansatzes:

$$\varphi(r) = \begin{cases} \frac{m_s^{\frac{1}{2}}}{(2\pi R^2)^{\frac{3}{4}}} e^{-\frac{r^2}{4R^2}}, & \text{Gaussian [27];} \\ \left(\frac{3m_s}{\pi^3 R^3}\right)^{\frac{1}{2}} \text{sech}\left(\frac{r}{R}\right), & \text{Sech [16];} \\ \left(\frac{m_s}{7\pi R^3}\right)^{\frac{1}{2}} \left(1 + \frac{r}{R}\right) e^{-\frac{r}{R}}, & \text{Exponential linear (EL) [16].} \end{cases}$$

normalized so that the density of the scalar field satisfies $\rho(r) \propto \varphi(r)^2$

The Hamiltonian

Kinetic energy

$$H_{\text{kin}} = \frac{1}{2m_a} \int d^3x \nabla \psi^* \cdot \nabla \psi$$

(Self) Gravitational energy

$$H_g = -\frac{Gm_a^2}{2} \int d^3x \int d^3x' \frac{|\psi(\vec{x}, t)|^2 |\psi(\vec{x}', t)|^2}{|\vec{x} - \vec{x}'|}$$

Self-interaction energy

$$H_i = \frac{\lambda}{16m_a^2} \int d^3x |\psi|^4$$

Static solution: $\psi(\vec{x}, t) = \Psi(r)e^{-i\epsilon t}$

$$\text{Ansatz: } \Psi(r) = \sqrt{\frac{3N}{\pi^3 R^3}} \operatorname{sech}\left(\frac{r}{R}\right)$$

[From Zihang Wang's slides]
formulae adapted from Schiappacasse
& Hertzberg, 18'

Mass-radius relation of dilute axion stars

